TRIGONOMETRY

Trigonometric Ratios and Identities

T-1 $1^\circ = 60'$ (60 min), $1' = 60''$ (60 sec)

$180^\circ = \pi$ rad $\Rightarrow 1^\circ = \frac{\pi}{180}$ rad

Sum of interior angles on convex polygon of n sides is $(n-2)\pi$ rad

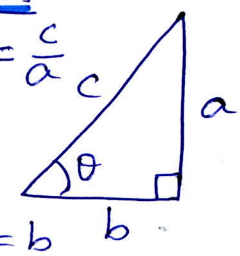
Sum of exterior angles on convex polygon of n sides is 2π rad.

T-2 Trigonometric functions

$\sin\theta = \frac{a}{c}$, $\operatorname{cosec}\theta = \frac{c}{a}$

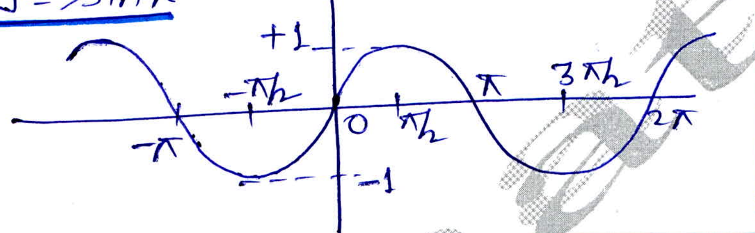
$\cos\theta = \frac{b}{c}$, $\sec\theta = \frac{c}{b}$

$\tan\theta = \frac{a}{b}$, $\cot\theta = \frac{b}{a}$

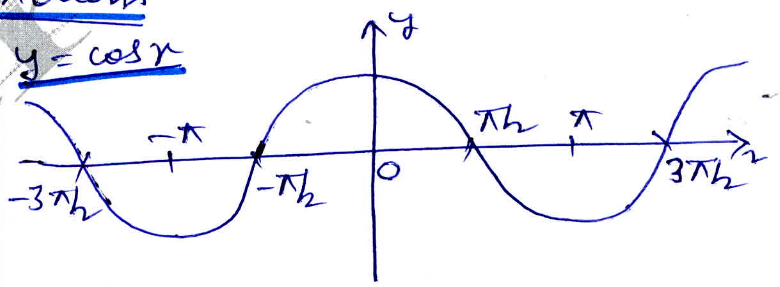


T-3 Graphs of trigonometric functions

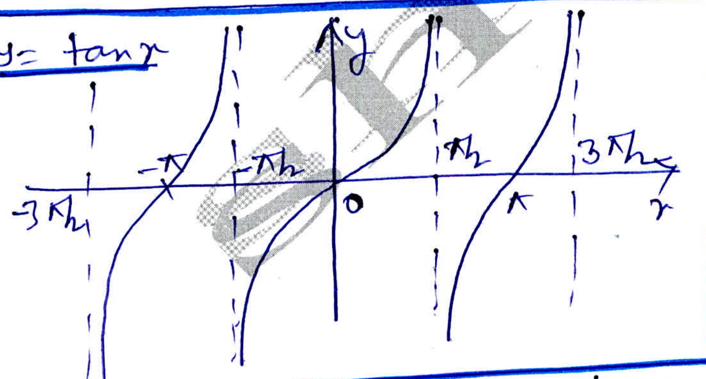
$y = \sin x$



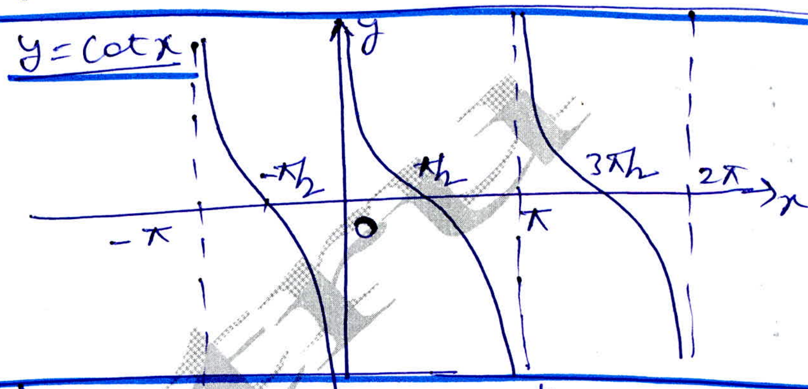
$y = \cos x$



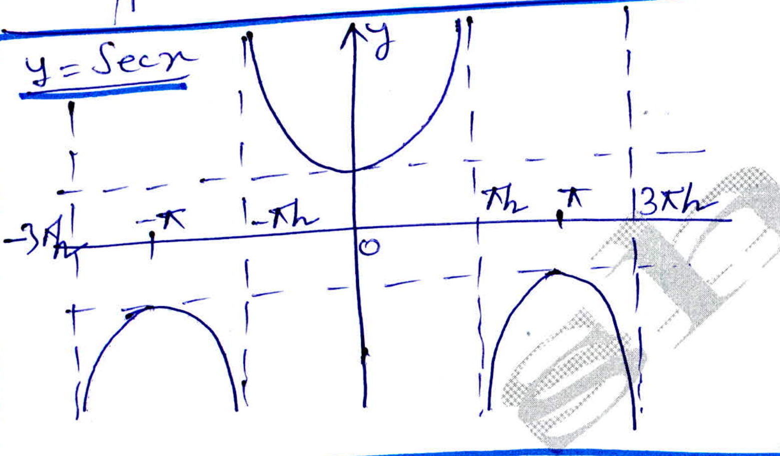
$y = \tan x$



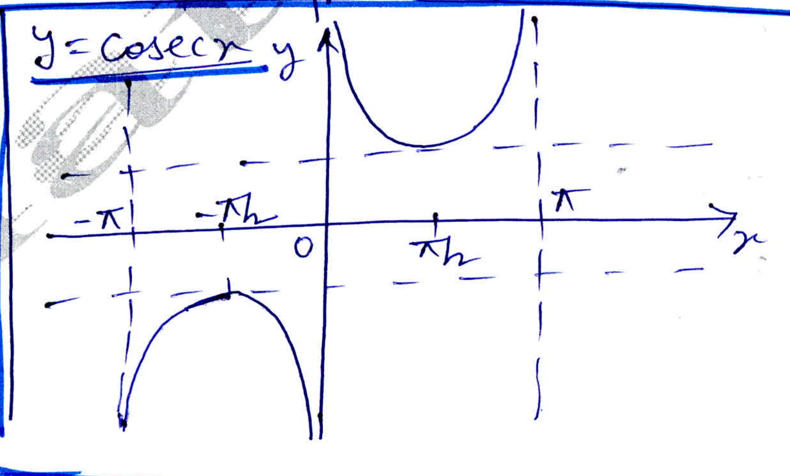
$y = \cot x$



$y = \sec x$



$y = \operatorname{cosec} x$



T-4 Difference and Sum of two angles

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

(ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

(iii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(iv) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$, (vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

(vii) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$, (viii) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Some More Results

a) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

b) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

c) $\sin(A+B+C) = \sin A \cos B \cos C + \cos A \sin B \cos C + \cos A \cos B \sin C - \sin A \sin B \sin C$

d) $\cos(A+B+C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$

e) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

T-5 Range of $f(\theta) = a\cos\theta + b\sin\theta$

$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

T-6 Product into Sum or difference

(i) $\sin A \cos B + \cos A \sin B = \sin(A+B)$

(ii) $\sin A \cos B - \cos A \sin B = \sin(A-B)$

adding (i) & (ii) we get

a) $2\sin A \cos B = \sin(A+B) + \sin(A-B)$, Similarly

b) $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

c) $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

d) $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

T-7 Sum or difference into Product

a) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

b) $\sin C - \sin D = 2\sin\dfrac{C-D}{2}\cos\dfrac{C+D}{2}$

c) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

d) $\cos C - \cos D = 2\sin\dfrac{C+D}{2}\sin\dfrac{D-C}{2}$

T-8 Multiple angles

(i) $\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

(iv) $\sin 3A = 3\sin A - 4\sin^3 A$

(v) $\cos 3A = 4\cos^3 A - 3\cos A$

(vi) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

(vii) $\tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$

(viii) $\cot 3A = \dfrac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$

T-9

(i) $\cos A \cos(60-A)\cos(60+A) = \dfrac{\cos 3A}{4}$

(ii) $\sin A \sin(60-A)\sin(60+A) = \dfrac{\sin 3A}{4}$

(iii) $\tan A \tan(60-A)\tan(60+A) = \tan 3A$

T-10 Values of trigonometric Ratios of standard angles

	$7\frac{1}{2}^\circ$	15°	18°	$22\frac{1}{2}^\circ$	36°	$67\frac{1}{2}^\circ$	75°
Sin	$\frac{\sqrt{8-2\sqrt{6}-2\sqrt{2}}}{4}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
Cos	$\frac{\sqrt{8+2\sqrt{6}+2\sqrt{2}}}{4}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
tan	$(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$	$2-\sqrt{3}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{2}+1$	$2+\sqrt{3}$
cot	$(\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$	$2+\sqrt{3}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{2}+1$	$\sqrt{1+\frac{2}{\sqrt{5}}}$	$\sqrt{2}-1$	$2-\sqrt{3}$

T-11 Sum N angles in A.P.

a) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin(\alpha+\overline{n-1}\beta) = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \times \sin\left[\alpha + \frac{(n-1)\beta}{2}\right]$

b) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+\overline{n-1}\beta) = \dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \times \cos\left[\alpha + \frac{(n-1)\beta}{2}\right]$

T-12 Conditional Identities

Standard identities in triangle $(A+B+C=\pi)$

(i) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(ii) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{C}{2}\tan\frac{B}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(iii) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

(iv) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(v) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(vi) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

T-13

$\cos A \cos 2A \cos 2^2A \cos 2^3A \cdots \cos 2^{n-1}A = \dfrac{\sin 2^n A}{2^n \sin A}$

T-14 Important Inequalities

(i) In ΔABC, $\tan A + \tan B + \tan C \geq 3\sqrt{3}$, A, B, C are acute angles

(ii) In ΔABC, $\cos A + \cos B + \cos C \leq 3/2$

(iii) In ΔABC, $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \leq \frac{1}{8}$

(iv) In an acute angled Δ, $\sec A + \sec B + \sec C \geq 6$

(v) In ΔABC, $\operatorname{cosec}\frac{A}{2} + \operatorname{cosec}\frac{B}{2} + \operatorname{cosec}\frac{C}{2} \geq 6$.

T-15 In ΔABC,

(i) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} - \cos^2\frac{C}{2} = 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

(ii) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

T-16 If $A+B+C = \frac{\pi}{2}$

(i) $\sin^2 A + \sin^2 B + \sin^2 C = 1 - 2\sin A \sin B \sin C$.

(ii) $\cos^2 A + \cos^2 B + \cos^2 C = 2 + 2\sin A \sin B \sin C$.

T-17 (i) $\tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$

(ii) In ΔABC, $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} \leq \frac{3}{2}$

(iii) In ΔABC, $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \geq 1$

TRIGONOMETRIC EQUATIONS

E-1

$\sin\theta = 0 \Rightarrow \theta = n\pi, \; n \in Z$

$\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, \; n \in Z$

$\tan\theta = 0 \Rightarrow \theta = n\pi, \; n \in Z$

$\sin\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}, \; n \in Z$

$\sin\theta = -1 \Rightarrow \theta = (4n-1)\frac{\pi}{2}, \; n \in Z$

$\cos\theta = 1 \Rightarrow \theta = 2n\pi, \; n \in Z$

$\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi, \; n \in Z$

$\cot\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, \; n \in Z$

E-2 General Solution

(i) $\sin\theta = \sin\alpha \Rightarrow n\pi + (-1)^n\alpha, \; n \in Z$

$\Rightarrow \sin\theta = k \Rightarrow \theta = n\pi + (-1)^n(\sin^{-1}k), \; n \in Z, \; k \in [-1,1]$

(ii) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, \; n \in Z$

$\Rightarrow \cos\theta = k \Leftrightarrow \theta = 2n\pi \pm (\cos^{-1}k), \; n \in Z, \; k \in [-1,1]$

(iii) $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, \; n \in Z$

$\Rightarrow \tan\theta = k \Rightarrow \theta = n\pi + (\tan^{-1}k), \; n \in Z, \; k \in R$

(iv) $\sin^2\theta = \sin^2\alpha$ or $\cos^2\theta = \cos^2\alpha$

$\Rightarrow \theta = n\pi \pm \alpha,\ n \in Z$

(v) $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha,\ n \in Z$

E-3 Solution of the eqⁿ of the form $a\cos\theta + b\sin\theta = c$

I- if $|c| > \sqrt{a^2+b^2}$, then no real solⁿ

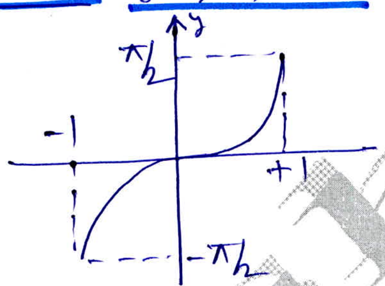
II- if $|c| \le \sqrt{a^2+b^2}$, then divide both sides of the eqⁿ by $\sqrt{a^2+b^2}$, then take $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$

eqⁿ will reduce to $\cos(\theta-\alpha) = \cos\beta$, where $\tan\alpha = \frac{b}{a}$

$\cos\beta = \frac{c}{\sqrt{a^2+b^2}}$

INVERSE TRIGONOMETRIC FUNCTIONS

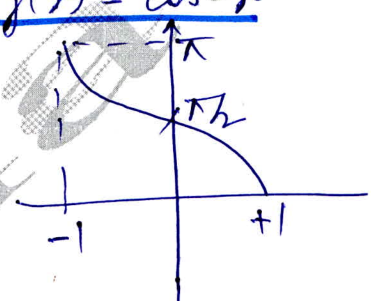
I-1 $f(x) = \sin^{-1}x$



$D_f : [-1, 1]$

$R_f : [-\frac{\pi}{2}, \frac{\pi}{2}]$

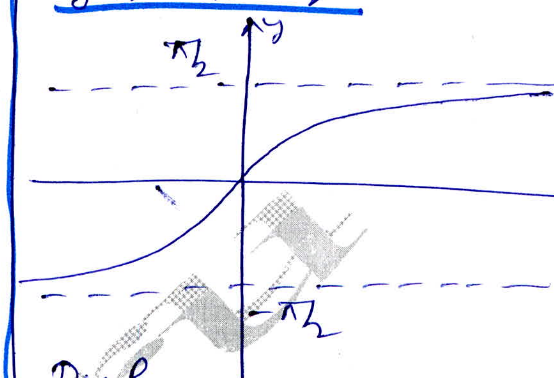
$f(x) = \cos^{-1}x$



$D_f : [-1, 1]$

$R_f : [0, \pi]$

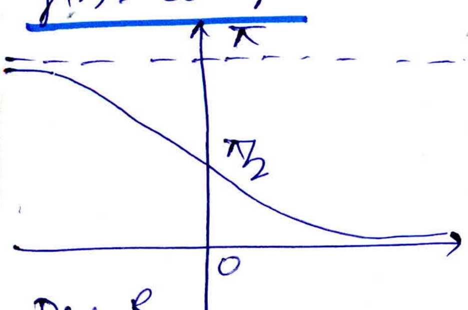
$f(x) = \tan^{-1}x$



$D_f : R$

$R_f : (-\frac{\pi}{2}, \frac{\pi}{2})$

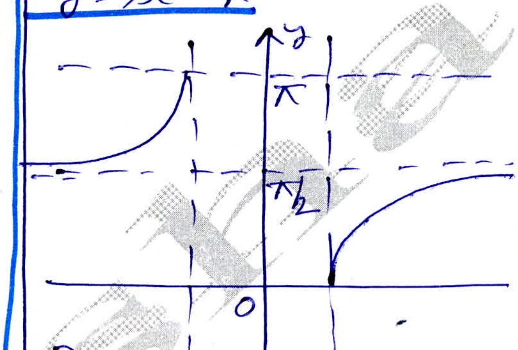
$f(x) = \cot^{-1}x$



$D_f : R$

$R_f : (0, \pi)$

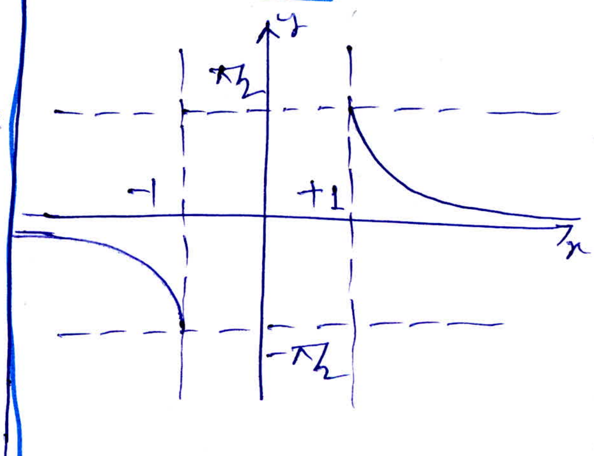
$y = \sec^{-1}x$



$D_f : (-\infty, -1] \cup [1, \infty)$

$R_f : [0, \pi] - \{\pi/2\}$

$y = \mathrm{cosec}^{-1}x$



I-2

(i) $\sin(\sin^{-1}x) = x$, for all $x \in [1, 1]$

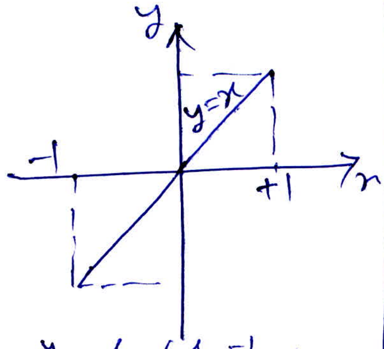
(ii) $\cos(\cos^{-1}x) = x$, for all $x \in [-1, 1]$

(iii) $\tan(\tan^{-1}x) = x$, for all $x \in R$

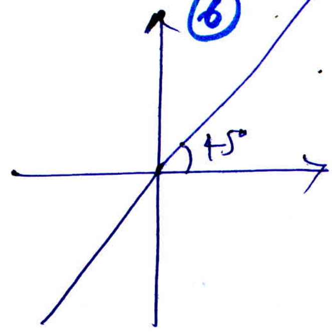
(iv) $\cot(\cot^{-1}x) = x$, for all $x \in R$

(v) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$ for all $x \in (-\infty, -1] \cup [1, \infty)$

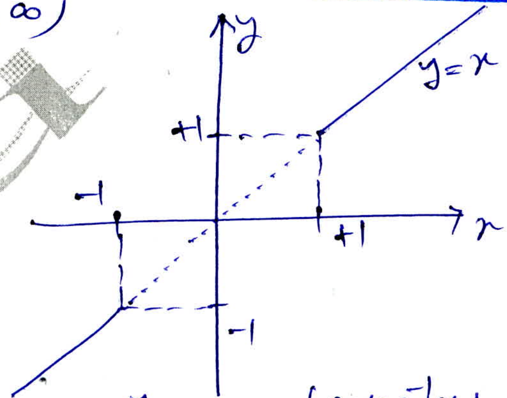
(vi) $\sec(\sec^{-1}x) = x$, for all $x \in (-\infty, -1] \cup [1, \infty)$



$y = \sin(\sin^{-1}x)$
$y = \cos(\cos^{-1}x)$



$y = \tan(\tan^{-1}x)$
$y = \cot(\cot^{-1}x)$



$y = \operatorname{cosec}(\operatorname{cosec}^{-1}x)$
$y = \sec(\sec^{-1}x)$

I-3

(i) $\sin^{-1}(\sin x) = x \quad \forall x \in [\frac{\pi}{2}, \frac{\pi}{2}]$

(ii) $\cos^{-1}(\cos x) = x \quad \forall x \in [0, \pi]$

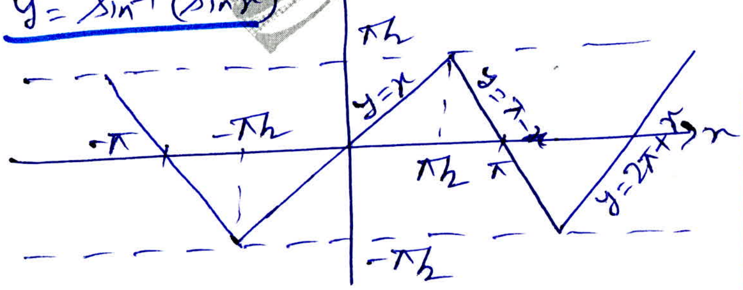
(iii) $\tan^{-1}(\tan x) = x \quad \forall x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

(iv) $\cot^{-1}(\cot x) = x \quad \forall x \in (0, \pi)$

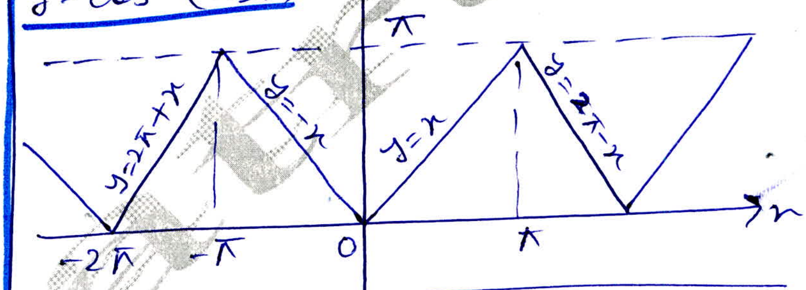
(v) $\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad \forall x \in [\frac{\pi}{2}, \frac{\pi}{2}] \sim \{0\}$

(vi) $\sec^{-1}(\sec x) = x \quad \forall x \in [0, \pi] \sim \{\frac{\pi}{2}\}$

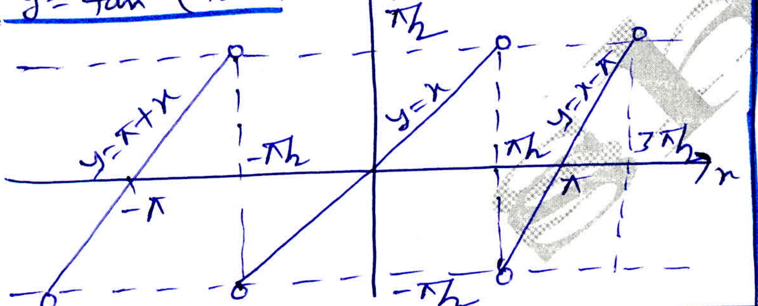
$y = \sin^{-1}(\sin x)$



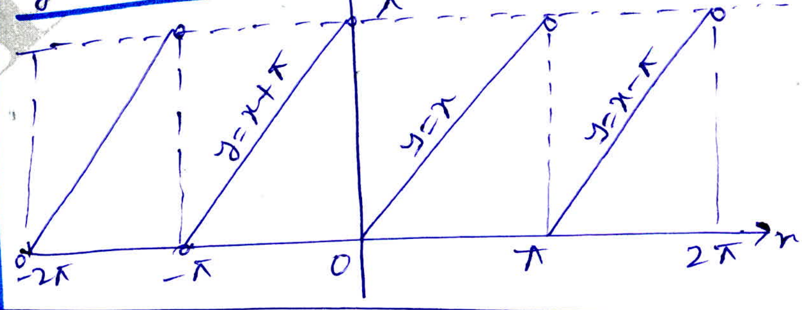
$y = \cos^{-1}(\cos x)$



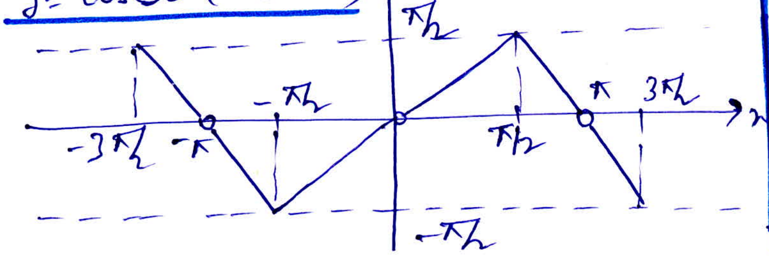
$y = \tan^{-1}(\tan x)$



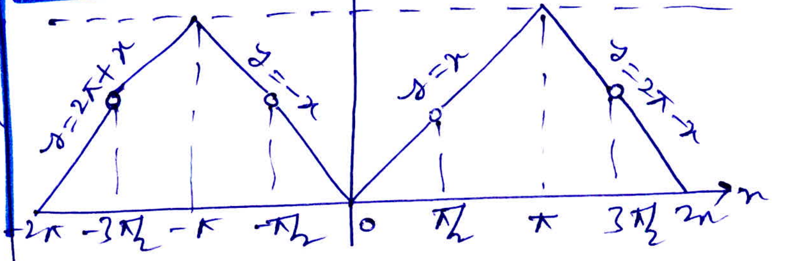
$y = \cot^{-1}(\cot x)$



$y = \operatorname{cosec}^{-1}(\operatorname{cosec} x)$



$y = \sec^{-1}(\sec x)$



I-4

(i) $\sin^{-1}(-x) = -\sin^{-1}x \quad \forall x \in [-1,1]$, (ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x, \forall x \in [-1,1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x, \forall x \in R$, (iv) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x), \forall x \in (-\infty,-1] \cup [1,\infty)$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x, \forall x \in (-\infty,-1] \cup [1,\infty)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x, \forall x \in R$.

I-5

(i) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}(x), \forall x \in (-\infty,-1] \cup [1,\infty)$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x, \forall x \in (-\infty,-1] \cup [1,\infty)$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x, & x > 0 \\ -\pi + \cot^{-1}x, & x < 0 \end{cases}$

I-6

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \forall x \in [-1,1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \forall x \in R$

(iii) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}, \forall x \in (-\infty,-1] \cup [1,\infty)$

I-7

(i) for $x > 0$,

$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

(ii) for $x > 0$,

$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$

(iii) for $x > 0$,

$\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\frac{1}{x}\right) = \sec^{-1}\sqrt{1+x^2} = \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

I-8

(i) $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } x > 0,\ y > 0\ \&\ xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } x < 0, y < 0\ \&\ xy > 1 \end{cases}$

(ii) $$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{if } xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) & \text{if } x>0,\ y<0 \ \& \ xy<-1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) & \text{if } x<0,\ y>0 \ \& \ xy<-1 \end{cases}$$

I-9

(i) $$\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), & x\geq 0,\ y\geq 0 \ \& \ x^2+y^2 \leq 1 \\ \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), & x\geq 0,\ y\geq 0 \ \& \ x^2+y^2 > 1 \end{cases}$$

(ii) $$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right),\ x\geq 0,\ y\geq 0$$

I-10

(i) $$2\sin^{-1}x = \begin{cases} \sin^{-1}\left(2x\sqrt{1-x^2}\right), & -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & x > \frac{1}{\sqrt{2}} \\ -\pi - \sin^{-1}\left(2x\sqrt{1-x^2}\right), & x < -\frac{1}{\sqrt{2}} \end{cases}$$

(ii) $$3\sin^{-1}x = \begin{cases} \sin^{-1}(3x-4x^3), & -\frac{1}{2} \leq x \leq \frac{1}{2} \\ \pi - \sin^{-1}(3x-4x^3), & x > \frac{1}{2} \\ -\pi - \sin^{-1}(3x-4x^3), & x < -\frac{1}{2} \end{cases}$$

I-11

(i) $$2\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } -1<x<1 & = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) & \text{if } x>1 & = \pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right) \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right) & \text{if } x<-1 & = -\pi - \sin^{-1}\left(\frac{2x}{1+x^2}\right) \end{cases}$$

(ii) $$3\tan^{-1}x = \begin{cases} \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & x < -\frac{1}{\sqrt{3}} \end{cases}$$

(iii) $$2\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & \text{if } 0 \leq x < \infty \\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) & \text{if } -\infty < x \leq 0 \end{cases}$$

I-12

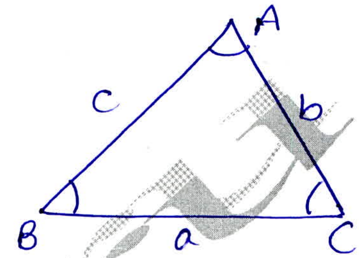
(i) $2\cos^{-1}x = \begin{cases} 2\pi - \cos^{-1}(2x^2-1), & \text{if } -1 \le x < 0 \\ \cos^{-1}(2x^2-1), & \text{if } 0 \le x \le 1 \end{cases}$

SOLUTIONS & PROPERTIES OF TRIANGLE

S-1 Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

where R is circumradius



S-2 Nepier's formula

(i) $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$

(ii) $\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$

(iii) $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

These formulae are useful when two sides and included angle are given

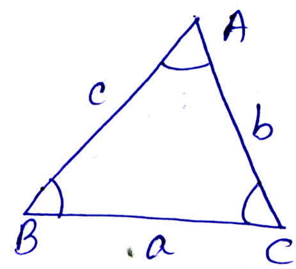
S-3 Cosine Rule

In ΔABC,

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

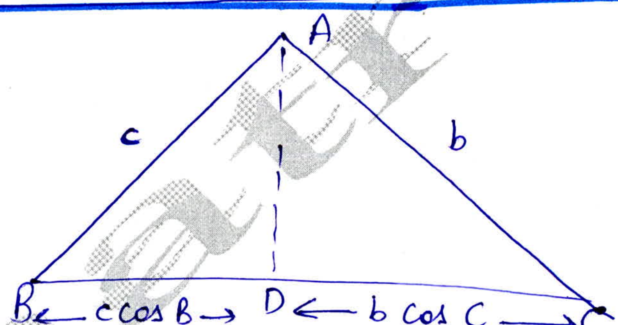


S-4 Projection formula

(i) $a = b\cos C + c\cos B$

(ii) $b = c\cos A + a\cos C$

(iii) $c = a\cos B + b\cos A$



S-5 Half-Angle formulae

(i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(ii) $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$

(iii) $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(i) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$

(ii) $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$

(iii) $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

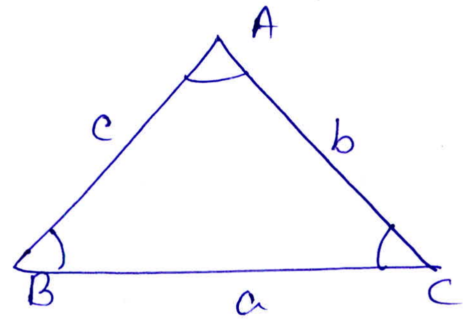
(i) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(ii) $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

(iii) $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

where $2s = a+b+c$ (perimeter of ΔABC, whose side lengths are a, b & c)

S-6 Area of triangle



(i) $\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$

(ii) $\Delta = \frac{abc}{4R}$, (R is circumradius)

(iii) $\Delta = 2R^2 \sin A \sin B \sin C$

(iv) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$

(v) $\Delta = rs$, where r is inradius

(vi) $\Delta = \frac{a^2 \sin B \sin C}{2 \sin A} = \frac{b^2 \sin C \sin A}{2 \sin B} = \frac{c^2 \sin A \sin B}{2 \sin C}$

S-7

(i) $\sin\alpha + \sin 2\alpha + \dots + \sin n\alpha = \frac{\sin\left(\frac{\alpha(n+1)}{2}\right) \sin\left(\frac{n\alpha}{2}\right)}{\sin\frac{\alpha}{2}}$

(ii) $\cos\alpha + \cos 2\alpha + \dots + \cos n\alpha = \frac{\cos\left((n+1)\frac{\alpha}{2}\right) \times \sin\left(\frac{\alpha n}{2}\right)}{\sin\frac{\alpha}{2}}$

(iii) $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{2\tan\theta/2}{1+\tan^2\theta/2}$

(iv) $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 1 - 2\sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$

(v) $\tan\theta = \frac{2\tan\theta/2}{1-\tan^2\theta/2}$, (vi) $\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$

(vii) $\sin\frac{\theta}{2} \pm \cos\frac{\theta}{2} = \sqrt{2}\sin\left(\frac{\pi}{4} \pm \theta\right) = \sqrt{2}\cos\left(\theta \mp \frac{\pi}{4}\right)$

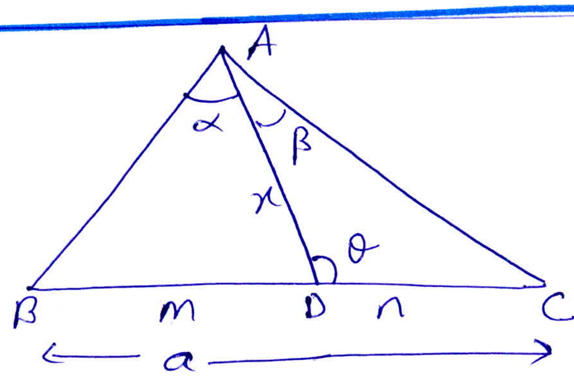
S-8 Clock

(i) The angle b/w two consecutive digits in a clock is $30°$.

(ii) The hour hand rotates through an angle of $30°$ in one hour or $\left(\frac{1}{2}\right)°$ in one minute.

(iii) The minute hand rotates through an angle of $6°$ in one minute.

S-9 m-n Theorems.



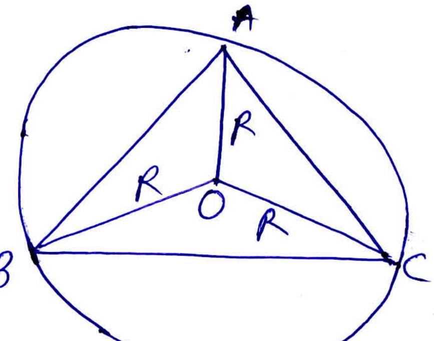
$BD : DC = m : n$, $|AD| = x$

(i) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

(ii) $(m+n)\cot\theta = n\cot B - m\cot C$

(iii) $(m+n)^2 x^2 = (m+n)(mb^2+nc^2) - mna^2$

(i) Circum Circle and Circum Centre (O)



a) $R = \dfrac{abc}{4\Delta} = \dfrac{a}{2\sin A} = \dfrac{b}{2\sin B} = \dfrac{c}{2\sin C}$

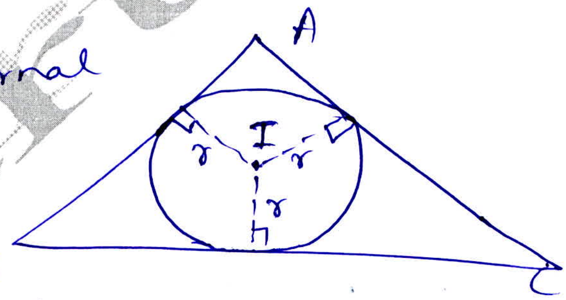
b) $\angle BOC = 2\angle BAC$

c) The circumcentre (O) may lie within, outside or upon any side of the Δ.

d) In a right angled Δ, the circumcentre is the mid point of hypotenuse.

(ii) Incircle and Incentre (I)

Point of intersection of the internal angle bisectors of a Δ is called the in-centre of the Δ.



a) $r = \dfrac{\Delta}{s}$

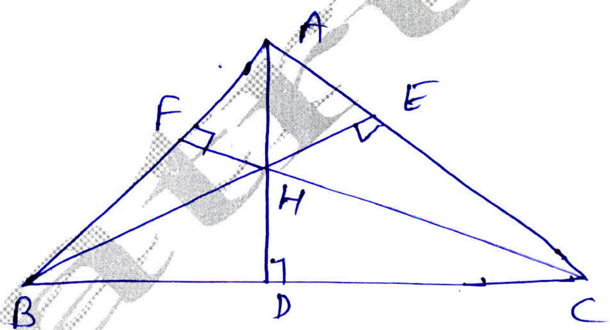
$= (s-a)\tan\dfrac{A}{2} = (s-b)\tan\dfrac{B}{2} = (s-c)\tan\dfrac{C}{2}$

$= 4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

b) $AI = \dfrac{r}{\sin\frac{A}{2}}$, $BI = \dfrac{r}{\sin\frac{B}{2}}$, $CI = \dfrac{r}{\sin\frac{C}{2}}$, (c) $\angle BIC = 90° + \dfrac{\angle A}{2}$

(iii) Orthocentre (H)

Pt of intersection of the altitudes of a Δ is called orthocentre (H)



a) Image of orthocentre (H) in any side of a Δ lies on the circumcircle.

b) $AH = 2R\cos A$, $BH = 2R\cos B$, $CH = 2R\cos C$

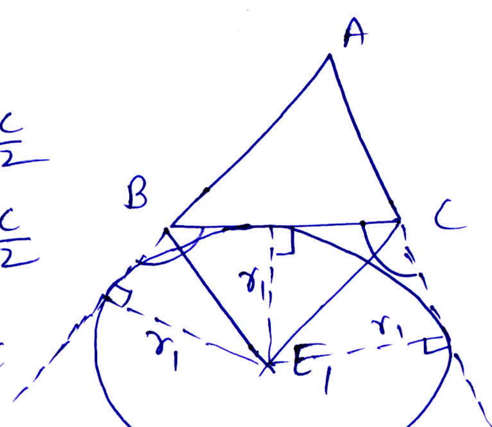
c) $HD = 2R\cos B\cos C$, $HE = 2R\cos A\cos C$, $HF = 2R\cos A\cos B$.

(iv) Escribed Circles and their radii

a) $r_1 = \dfrac{\Delta}{s-a} = s\tan\dfrac{A}{2} = 4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

b) $r_2 = \dfrac{\Delta}{s-b} = s\tan\dfrac{B}{2} = 4R\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$

c) $r_3 = \dfrac{\Delta}{s-c} = s\tan\dfrac{C}{2} = 4R\cos\dfrac{A}{2}\cos\dfrac{B}{2}\sin\dfrac{C}{2}$



d) $r_1+r_2+r_3-r=4R$, e) $r_1r_2+r_2r_3+r_3r_1=s^2=\frac{r_1r_2r_3}{r}$

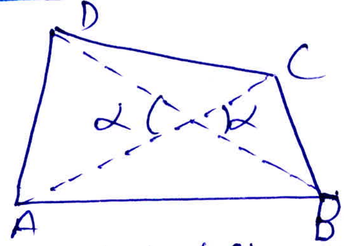
f) $\frac{r_1r_2r_3}{s}=\Delta$, g) for right angled Δ, $r_1=r_2+r_3+r$

S-11

In ΔABC, $R \geqslant 2r$, ($R=2r$ for equilateral Δ)

S-12

Area of quadrilateral

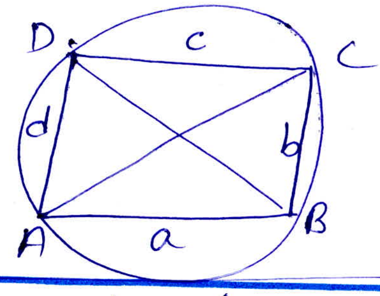


$S=\frac{1}{2}\, BD \times AC \times \sin\alpha$

$=\frac{1}{2}$ (product of the diagonals) × (sine of included angle)

S-13 c) Area $S=\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $2s=a+b+c+d$

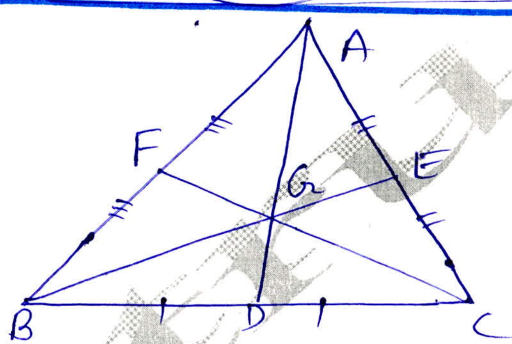
Cyclic Quadrilateral (which can be inscribed in a circle)



a) $\angle A+\angle C=\angle B+\angle D=180°$

b) $AB \times CD + AD \times BC = AC \times BD$
(Ptolemy's theorem)

S-14 Centroid of Triangle (G)



Pt of concurrency of three medians is called the centroid.

a) Centroid (G) of a triangle is situated on line joining its circumcentre (O) and orthocentre (H) and divide this line in the ratio 1:2

O — G —— H
1 : 2

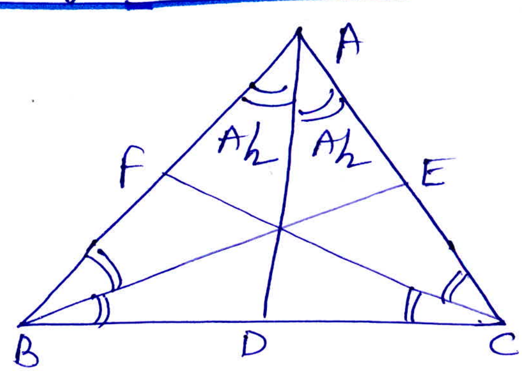
b) $AB^2+AC^2=2(AD^2+BD^2)$ (Apollonius Theorem)

S-15 Length of the Bisectors of the angles of a triangle

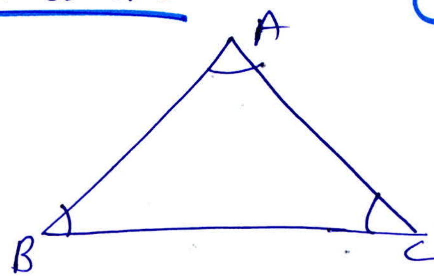
(i) $AD=\frac{2bc}{b+c}\cos\frac{A}{2}$

(ii) $BE=\frac{2ac}{a+c}\cos\frac{B}{2}$

(iii) $CF=\frac{2ab}{a+b}\cos\frac{C}{2}$



S-16 Distance b/w In-Centre and Ex-centre



$$II_1 = \frac{r}{\sin\frac{B}{2}\sin C/2}$$

$$II_2 = \frac{r}{\sin(A/2)\sin(C/2)}, \quad II_3 = \frac{r}{\sin(A/2)\sin(B/2)}$$

S-17 Distance b/w Ex-centres

$$I_1I_2 = 4R\cos\frac{C}{2}, \quad I_2I_3 = 4R\cos\frac{A}{2}, \quad I_3I_1 = 4R\cos\frac{B}{2}$$

S-18 Distance b/w the Circumcentre and the Orthocentre

$$OH = R\sqrt{1-8\cos A\cos B\cos C},$$ where O: Circumcentre, H: Orthocentre

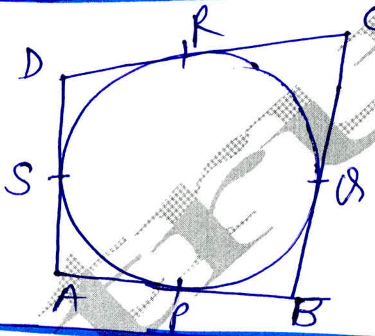
S-19 Distance of the circumcentre from Incentre and Excentres

$$OI = \sqrt{R^2-2Rr}, \quad OI_1 = \sqrt{R^2+2Rr_1}, \quad OI_2 = \sqrt{R^2+2Rr_2}, \quad OI_3 = \sqrt{R^2+2Rr_3}$$

Area of $\Delta ABC = \frac{4}{3}$(area of triangle whose sides are of length equal to medians of the ΔABC)

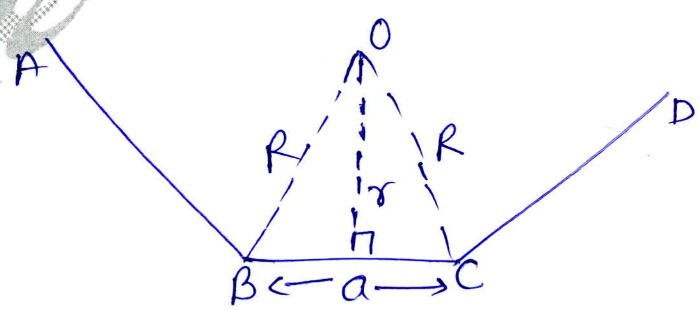
S-20 An Imp Result-

$AB + CD = AP + BP + CR + DR$
$= AS + BQ + CQ + DS = AD + BC$



S-21 Regular Polygon (n sided)

⇒ In the regular polygon, the circumcentre and the in-centre are the same.



⇒ Sum of internal angles $= (n-2)\pi$

⇒ each internal angle $= \frac{(n-2)\pi}{n}$

⇒ $R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}, \quad r = \frac{a}{2}\cot\frac{\pi}{n}$

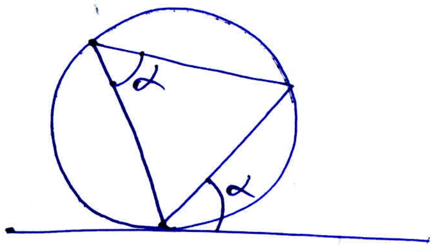
⇒ Area of the regular polygon $= \frac{1}{4}na^2\cot\frac{\pi}{n}$

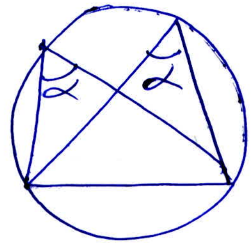
$= \frac{1}{2}nR^2\sin\left(\frac{2\pi}{n}\right)$

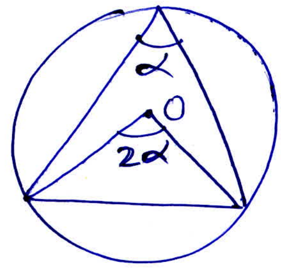
$= nr^2\tan\left(\frac{\pi}{n}\right)$

where a is the side of polygon.

S-22







O: centre