INDEFINITE INTEGRATION

I-1 $\frac{d}{dx} g(x) = f(x) \Rightarrow \int f(x)\,dx = g(x) + c$

a) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$

b) $\int \frac{1}{x}\,dx = \log|x| + c, \quad x \neq 0$

c) $\int e^x\,dx = e^x + c$

d) $\int a^x\,dx = \frac{a^x}{\log_e a} + c$

e) $\int \sin x\,dx = -\cos x + c$

f) $\int \cos x\,dx = \sin x + c$

g) $\int \sec^2 x\,dx = \tan x + c$

h) $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

i) $\int \sec x \tan x\,dx = \sec x + c$

j) $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

k) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$ or $-\cos^{-1} x + c$

l) $\int \frac{dx}{1+x^2} = \tan^{-1} x$ or $-\cot^{-1} x + c$

m) $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1} x$ or $-\operatorname{cosec}^{-1} x + c$

I-2 If $\int f(x)\,dx = F(x) + c$, then $\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$

I-3 $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

I-4 $\int (f(x))^n \cdot f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + c$

I-5 a) $\int \tan x\,dx = \ln|\sec x| + c$

b) $\int \cot x\,dx = \ln|\sin x| + c$

c) $\int \sec x\,dx = \ln|\sec x + \tan x| + c = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

d) $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + c = \ln\left|\tan \frac{x}{2}\right| + c$

I-6 $\int (\sin^m x \cos^n x)\,dx, \quad m, n \in N$

a) If one of them is odd, then substitute for the term of even power

b) If both are odd, substitute either of them

c) If both are even, use trigonometric identities only

d) If m & n are rational nos. & $\frac{m+n-2}{2}$ is −ve int, put $\tan x = p$ or $\cot x = p$.

I7 $\int \frac{dx}{\text{Quadratic}}$

a) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

b) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$

c) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c$

I-8 $\int \frac{dx}{a+b\sin x+c\cos x}$

Write $\sin x$ and $\cos x$ in terms of $\tan\frac{x}{2}$ and substitute t for $\tan\frac{x}{2}$

$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$

$\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

I-9 $\int \frac{dx}{a\cos^2x+b\sin^2x}$, $\int \frac{dx}{a+b\sin^2x}$, $\int \frac{dx}{a+b\cos^2x}$

$\int \frac{dx}{(a\sin x+b\cos x)^2}$, $\int \frac{dx}{a+b\sin^2x+b\cos^2x}$

$\Rightarrow$ divide both the N^r and D^r by $\cos^2x$, put $\tan x = t$

I-10 $\int \frac{p\cos x+q\sin x+r}{a\cos x+b\sin x+c}\,dx$

$\Rightarrow$ express N^r as λ(denominator)$+\mu$(differentiation of D^r)$+\gamma$ find λ, μ and γ by comparing coeff of $\sin x$, $\cos x$ and constant term and split the integral into the sum of three integrals.

I-11 Biquadratic form $\left(\frac{\text{Quadratic}}{\text{Biquadratic}}\right)$

generally, create $\left(1\pm\frac{1}{x^2}\right)$ in the N^r by dividing N^r and D^r by x^2 then substitute $x\pm\frac{1}{x}=t$

I-12 Some Standard Substitution

$a^2+x^2 \rightarrow x = a\tan\theta$ or $a\cot\theta$

$a^2-x^2 \rightarrow x = a\sin\theta$ or $a\cos\theta$

$x^2-a^2 \rightarrow x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$

$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}} \rightarrow x = a\cos 2\theta$

$\sqrt{\frac{x-\alpha}{\beta-x}}$ or $\sqrt{(x-\alpha)(x-\beta)}$

$\rightarrow x = \alpha\cos^2\theta+\beta\sin^2\theta$

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I-13 $\int \frac{dx}{\sqrt{\text{quadratic}}}$

a) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$

b) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|\left(x+\sqrt{x^2+a^2}\right)\right|+c$

c) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|\left(\sqrt{x^2-a^2}+x\right)\right|+c$

I-16

$\int e^x\left(f(x)+f'(x)\right)dx$

$= e^x f(x)+c$

I-14 $\int \frac{px+q}{ax^2+bx+c}dx$ or $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

$\Rightarrow$ express $px+q = \lambda \frac{d}{dx}(ax^2+bx+c)+\mu \Rightarrow px+q = \lambda(2ax+b)+\mu$

find λ & μ and replace $px+q$.

I-15 Integration by Parts

u & v are two functions of x, then

$\int uv\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$

or

$\int \mathrm{I}\cdot\mathrm{II}\,dx = \mathrm{I}\int \mathrm{II}\,dx - \int\left(\frac{d}{dx}\mathrm{I}\int \mathrm{II}\,dx\right)dx$

The funⁿ on the left is always chosen as the first funⁿ according to I L A T E

I → Inverse, L → Log, A → Algebraic, T → trigono, E → Exponent

I-17 a) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

b) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(b\sin bx + a\cos bx)$

I-18 Integration by Partial fractions

While using the method of partial fractions, we must have the degree of polynomial in the N^r always less than that of D^r. If it is not so, then we carry out the division and reduce the degree of N^r to less than that of the D^r.

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I19 $\int \sqrt{\text{Quadratic}}\, dx$

a) $\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + c$

b) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + c$

c) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

I-20 $\int \text{Linear}\sqrt{\text{Quadratic}}\, dx$

$\Rightarrow$ Substitute Linear $= m\,(\text{Quadratic})' + n$.

I-21 $\int \frac{1}{L_1\sqrt{L_2}}\,dx$, $\int \frac{L_1}{\sqrt{L_2}}\,dx$, $\int \frac{\sqrt{L_2}}{L_1}\,dx$ ($L \Rightarrow$ Linear)

$\Rightarrow$ substitute t^2 for L_2

I-22 $\int \frac{1}{\text{Linear}\sqrt{\text{Quadratic}}}\,dx$, substitute for $\frac{1}{t} =$ Linear

I-23 $\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$

substitute for $x = \frac{1}{t}$, then the integrand reduces to $\int \frac{t\,dt}{pt^2+q\sqrt{rt^2+s}}$ and then substitute u^2 for (rt^2+s).

I-24 $\int \frac{dx}{\text{Quadratic}\sqrt{\text{linear}}}$, put linear $= t^2$

I-25 $\int x^m (a+bx^n)^p\, dx$

a) If $p \in N$, expand using binomial and integrate

b) If $p \in I^-$, write $x = t^k$, where $k = \text{LCM}(m, n)$

c) If $\frac{m+1}{n}$ is an integer and $p \to$ fraction, put $a+bx^n = t^k$, where k is denominator of the fraction p.

d) If $\left(\frac{m+1}{n}+p\right)$ is an integer and $p \in$ fraction, put $(a+bx^n) = t^k \cdot x^n$, where k is denominator of the fraction p.

I-26 $\int \frac{dx}{(x-k)^r\sqrt{ax^2+bx+c}}$, where $r \geq 2$, & $r \in I$ substitute $x-k = \frac{1}{t}$

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I-27 $\int \frac{dx}{(a+b\cos x)^2}$, $\int \frac{dx}{(a+b\sin x)^2}$, $\int \frac{(a+b\sin x)\,dx}{(b+a\sin x)^2}$ (5)

$\Rightarrow$ let $A = \frac{\sin x}{a+b\cos x}$ or $A = \frac{\cos x}{a+b\sin x}$ according to the integral

$\Rightarrow$ find $\frac{dA}{dx}$ and express it in terms of $\frac{1}{a+b\cos x}$ or $\frac{1}{a+b\sin x}$

$\Rightarrow$ Now integrate both sides of the expression obtained in $\frac{dA}{dx}$.

I-28 Non-Integrable expressions

$\int \frac{\sin x}{x}$, $\int \frac{\cos x}{x}$, $\int \frac{1}{\log x}$, $\int \sqrt{\sin x}$, $\int \sin(x^2)$, $\int \cos(x^2)$

$\int x\tan x$, $\int e^{-x^2}$, $\int e^{x^2}$, $\int \sqrt[3]{1+x^2}$, $\int \sqrt{1+x^3}\,dx$.

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NOTES

DEFINITE INTEGRATION

D-1 Let f be a function of x defined in the closed interval $[a,b]$ and $\phi(x)$ be another function, such that $\phi'(x)=f(x)$ for all x in the domain of f, then

$$\int_a^b f(x)dx = [\phi(x)+c]_a^b = \phi(b)-\phi(a)$$

D-2 Definite Integral as the limit of a sum

$$\lim_{n\to\infty}\sum_{r=0}^{n-1} h f(a+rh) = \lim_{n\to\infty}\sum_{r=1}^{n} h f(a+rh) = \int_a^b f(x)dx \quad (b=a+nh)$$

put $a=0$ & $b=1$.

$$\Rightarrow \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$$

In general, $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{kn} f\left(\frac{r}{n}\right) = \int_0^k f(x)dx$

D-3 Some Important Series

a) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$

b) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin(\alpha+\overline{n-1}\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\frac{2\alpha+(n-1)\beta}{2}\right)$

c) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+\overline{n-1}\beta) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cdot\cos\left(\frac{2\alpha+(n-1)\beta}{2}\right)$

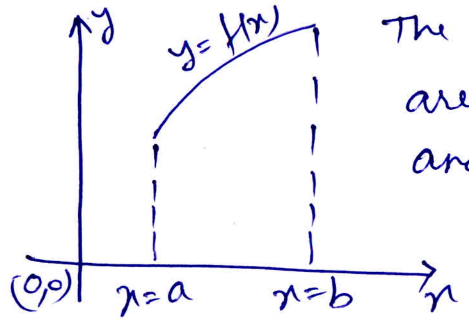
d) $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}-\frac{1}{6^2}+\cdots = \frac{\pi^2}{12}$

e) $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots = \frac{\pi^2}{6}$

f) $1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots = \frac{\pi^2}{8}$

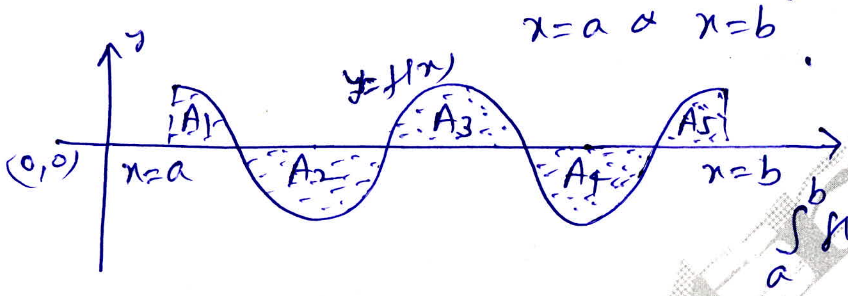
g) $\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\cdots = \frac{\pi^2}{24}$

D-4 Geometrical Interpretation of the Definite Integral



The integral $\int_a^b f(x)dx$ is numerically equal to the area bounded by the curve $y=f(x)$, the x-axis and the ordinates $x=a$ and $x=b$.

In general, $\int_a^b f(x)dx$ represents an algebraic sum of areas of the region bounded by the curve $y=f(x)$, the x-axis and the ordinates $x=a$ & $x=b$. The area above the x-axis are taken +ve, while those below the x-axis are taken −ve.



$$\int_a^b f(x)dx = A_1 - A_2 + A_3 - A_4 + A_5$$

D-5 Properties of Definite Integrals

(i) $\int_a^b f(x)dx = \int_a^b f(t)dt$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where c may lie inside or outside the interval $[a,b]$

(iv) $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$.

(v) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$ and

$$\int_0^{2a} f(x)dx = \begin{cases} 0 & \text{if } f(2a-x) = -f(x) \\ 2\int_0^a f(x)dx & \text{if } f(2a-x) = f(x) \end{cases}$$

(vi) $\int_0^{2a} f(x)\,dx = \int_0^a \{f(a-x) + f(a+x)\}\,dx$

(vii) $$\int_{-a}^a f(x)dx = \begin{cases} 0, & \text{if } f(x) \text{ is odd fun}^n\text{, ie. } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & \text{if } f(x) \text{ is even fun}^n\text{, ie } f(-x) = f(x) \end{cases}$$

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(viii) If $f(t)$ is an odd fun^n, then $\phi(x) = \int_0^x f(t)\,dt$ is an even fun^n

(ix) $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$, where T is the period of $f(x)$ & $n \in I$
$(f(x+T) = f(x))$

(x) $\int_a^{a+nT} f(x)\,dx = n\int_0^T f(x)\,dx$, $f(x+T) = f(x)$, $n \in I$

(xi) $\int_{mT}^{nT} f(x)\,dx = (n-m)\int_0^T f(x)\,dx$, $f(x+T) = f(x)$, $m, n \in I$

(xii) $\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)\,dx$, $f(x+T) = f(x)$, $n \in I$

Important Result: $\int_0^{\pi/2} \log \sin x\,dx = \int_0^{\pi/2} \log \cos x\,dx = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$

D-6 Leibnitz's Rule

a) If f is a continuous function on $[a,b]$ and $g(x)$ and $h(x)$ are differentiable fun^n of x whose values lie in $[a,b]$, then

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\frac{d}{dx}(h(x)) - f(g(x))\frac{d}{dx}(g(x))$$

b) If $F(t) = \int_a^b g(x,t)\,dx$, then $\frac{dF}{dt} = \int_a^b \frac{\partial g(x,t)}{\partial t}\,dx$, where $\frac{\partial g}{\partial t}$ represents the derivative of g with respect to t keeping x constant.

D-7 Inequalities

a) If $g(x) \le f(x) \le h(x)$ $\forall x \in [a,b]$ then

$$\int_a^b g(x)\,dx \le \int_a^b f(x)\,dx \le \int_a^b h(x)\,dx$$

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b) If m is the global minima and M is the global maxima of the function $f(x)$ on the interval $[a,b]$ then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

c) $$\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$$

d) If $f^2(x)$ and $g^2(x)$ are integrable on the interval $[a,b]$ then $$\left|\int_a^b f(x)\,g(x)\,dx\right| \leq \sqrt{\left(\int_a^b f^2(x)\,dx\right)\left(\int_a^b g^2(x)\,dx\right)}$$

D-8

$$\int_a^b f(x)\,d(g(x)) + \int_a^b g(x)\,d(f(x)) = f(b)g(b) - f(a)g(a)$$

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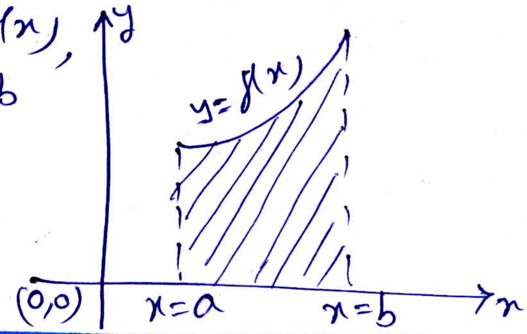
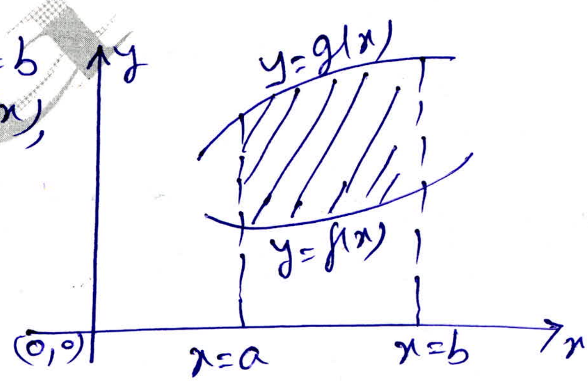
AREA

A-1 Area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ & $x = b$ $(b > a)$ is given by

$$A = \int_a^b f(x)\,dx = \int_a^b y\,dx$$

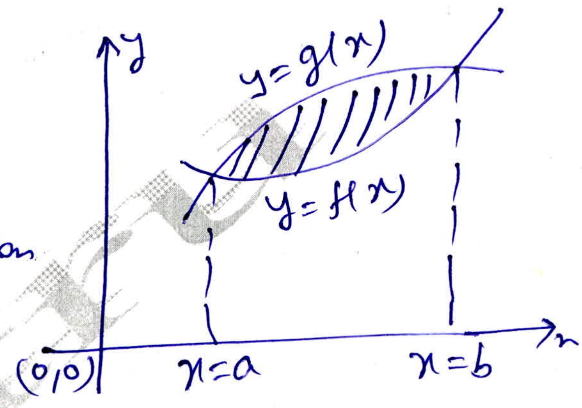


A-2 Area bounded by st lines $x = a$, $x = b$ $(a < b)$ and the curves $y = f(x)$, $y = g(x)$, provided $f(x) \le g(x)$ $(a \le x \le b)$ is given by $A = \int_a^b [g(x) - f(x)]\,dx$



A-3 When two curves intersect, $y = f(x)$ and $y = g(x)$, the bounded area is given by $A = \int_a^b [g(x) - f(x)]\,dx$, where a & b are the roots of the equation

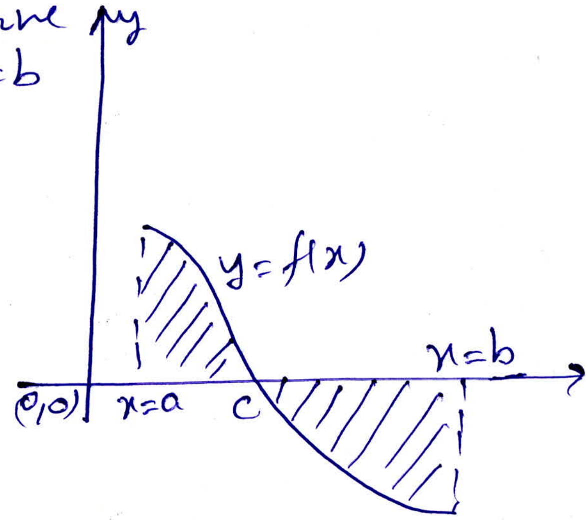
$$f(x) = g(x)$$



A-4 If the curve crosses the x-axis at c, then the area bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ $(b > a)$ is given by

$$A = \left|\int_a^c f(x)\,dx\right| + \left|\int_c^b f(x)\,dx\right|$$

$$= \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$$



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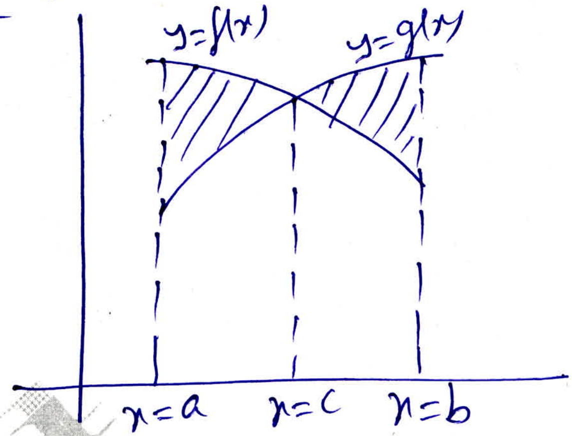
A-5 The area bounded by $y=f(x)$ & $y=g(x)$ $(a \le x \le b)$ when they intersect at $x=c \in (a,b)$ is given by

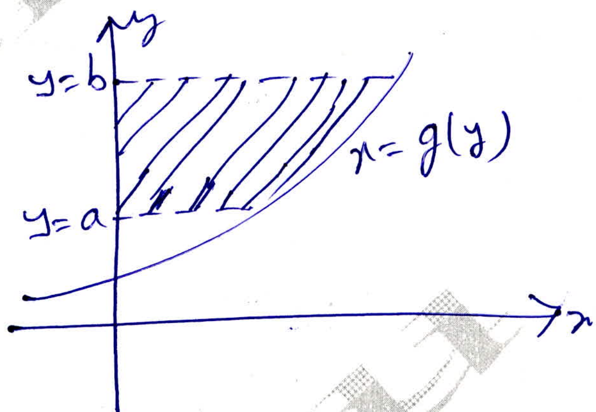
$$A = \int_a^b |f(x)-g(x)|\,dx$$

$$= \int_a^c (f(x)-g(x))\,dx + \int_c^b (g(x)-f(x))\,dx$$



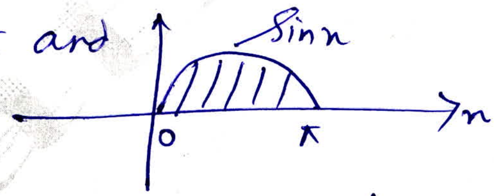
A-6 <u>Area bounded by the curves while Integrating along y-axis</u>

$$A = \int_a^b x\,dy$$

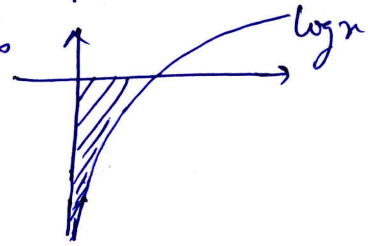


A-7 <u>Some Standard Areas</u>

a) Area bounded by $y=\sin x$, $0<x\le \pi$ and x axis is 2 sq units



b) Area bounded by $y=\log_e x$, $y=0$, $x=0$ is 1 sq. unit.



c) Area of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

d) Area bounded by $y^2=4ax$ & $x^2=4by$ $(a>0, b>0)$ is $\frac{16ab}{3}$.

e) Area bounded by $y^2=4ax$ & $y=mx$ is $\frac{8a^2}{3m^3}$.

A-8 If $y=f(x)$ is monotonic fn in (a,b), then the area bounded by $x=a$, $x=b$, $y=f(x)$ & $y=f(c)$ $(c \in (a,b))$ is minimum when $c=\frac{a+b}{2}$

DIFFERENTIAL EQUATION

E-1 An equation that involves independent and dependent variables and the derivatives of the dependent variable w.r.t independent variable is called a differential eqⁿ.

eg: $\frac{dy}{dx} = x^2 \log x$, $y = x\frac{dy}{dx} + a$.

E-2 The order of a DE is the order of the highest order differential coefficient occurring in it.

The degree of a DE which is expressed or can be expressed as a polynomial in the derivatives is the degree of the highest order derivative occurring in it after it has been expressed in a form free from radicals and fractions as far as derivatives are concerned.

E-3 Order of the DE will be equal to the number of independent parameters in the family of curves.

E-4 Solution of a DE

a) Variable Separation : $f(x)dx + g(y)dy = 0$.

$$\Rightarrow \int f(x)dx + \int g(y)dy = C$$

b) Eqⁿ reducible to V-S type : $\frac{dy}{dx} = f(ax+by+c)$

$$\Rightarrow ax+by+c = t$$

c) Homogeneous Eqⁿ : $\frac{dy}{dx} = \frac{f(x,y)}{\phi(x,y)}$, where $f(x,y)$ and $\phi(x,y)$ are homogeneous funⁿs of x & y, and of same degree.

$\Rightarrow \frac{dy}{dx} = g\left(\frac{y}{x}\right) \Rightarrow y = tx \Rightarrow$ eqⁿ will become Variable-Separation form

d) Eqⁿ reducible to Homogeneous form :

$$\frac{dy}{dx} = \frac{ax+by+c}{Ax+By+C}$$

if $aB \neq Ab$ & $A+b \neq 0$, $\Rightarrow x = X+h$, $y = Y+k \Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$

$$\Rightarrow \frac{dY}{dX} = \frac{aX+bY+(ah+bk+c)}{AX+BY+(Ah+Bk+C)}$$

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$\Rightarrow ah + bk + c = 0$, $Ah + Bk + C = 0 \Rightarrow$ find h & k.

$\Rightarrow \frac{dy}{dx} = \frac{ax+by}{Ax+By} \Rightarrow y = Xt$

If $aB = Ab$, $\Rightarrow ax+by = t$

If $A+b=0$, simply cross multiply and substitute $d(xy)$ for $xdy + ydx$.

E-5 Linear Differential Eq^n

a) $\frac{dy}{dx} + Py = Q$ where P & Q are fun^n of x alone

$I.F. = e^{\int P dx}$

sol^n: $y.(IF) = \int (Q.(IF))\,dx + C$

b) $\frac{dx}{dy} + P_1 x = Q_1$ where P_1 & Q_1 are fun^n of y alone

$I.F = e^{\int P_1 dy}$.

sol: $x.(IF) = \int (Q_1 (IF))\,dy + C$

E-6 Bernoulli's Eq^n

$\frac{dy}{dx} + Py = Qy^n \Rightarrow \frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q \Rightarrow \frac{1}{y^{n-1}} = t$

$\Rightarrow \frac{1}{y^n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dt}{dx} \Rightarrow \frac{dt}{dx} + (1-n)tP = Q(1-n) \rightarrow$ Linear Differential Eq.

E-7 General form of Variable Separation

(i) $xdy + ydx = d(xy)$

(ii) $\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$

(iii) $\frac{ydx - xdy}{y^2} = d\left(\frac{x}{y}\right)$

(iv) $\frac{xdy + ydx}{xy} = d(\ln xy)$

(v) $\frac{xdy - ydx}{xy} = d\left(\ln \frac{y}{x}\right)$

(vi) $\frac{xdy - ydx}{x^2+y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$

(vii) $\frac{xdx + ydy}{\sqrt{x^2+y^2}} = d\left[\sqrt{x^2+y^2}\right]$

(viii) $xdx + ydy = \frac{1}{2}d(x^2+y^2)$

(ix) $\frac{xdy + ydx}{x^2y^2} = d\left(-\frac{1}{xy}\right)$

(x) $\frac{dx+dy}{x+y} = d(\ln(x+y))$

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E-8 Orthogonal Trajectory

Any curve, which cuts every member of a given family of curves at right angles, is called an orthogonal trajectory of the family.

Let $f(x, y, c) = 0$ be the eqn of family of curves.

$\Rightarrow$ differentiate wrt x and eliminate 'C'.

$\Rightarrow$ substitute $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ $\Rightarrow$ This will give the DE of orthogonal trajectory.

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