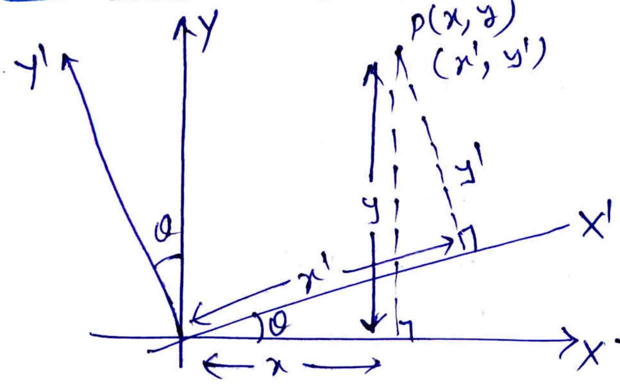
COORDINATE GEOMETRY

Straight Lines

S-1 Rotation of Axes



$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

and

$x' = x\cos\theta + y\sin\theta$, $y' = -x\sin\theta + y\cos\theta$

	$x\downarrow$	$y\downarrow$
$x'\rightarrow$	$\cos\theta$	$\sin\theta$
$y'\rightarrow$	$-\sin\theta$	$\cos\theta$

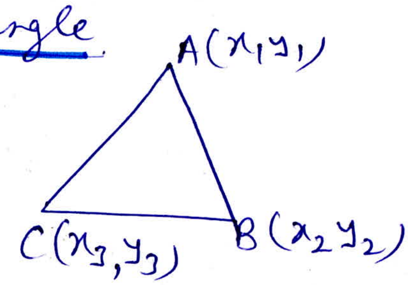
S-2 Distance formula

P $(x_1 y_1)$ Q $(x_2 y_2)$

$|PQ| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

S-3 Area of triangle

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$



or

$$\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

or

Stair Method

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\left[(x_1y_2+x_2y_3+x_3y_1)-(x_2y_1+x_3y_2+x_1y_3)\right]$$

S-4

⇒ If three points A, B and C are collinear, then area of ΔABC is zero or vice-versa

⇒ If the vertices of a triangle have rational coordinates, then the triangle can not be equilateral.
or if area is a rational no., then the Δ can not be equilateral

S-4 Area of Polygon

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \\ x_n & y_n \\ x_1 & y_1 \end{vmatrix}$$

$$\Delta = \frac{1}{2}\left[(x_1y_2+x_2y_3+\cdots+x_ny_1) - (x_2y_1+x_3y_2+\cdots+x_1y_n)\right]$$

Points should be taken in cyclic order.

S-5 - Section Formula

Internal division

$\leftarrow m \rightarrow \leftarrow n \rightarrow$

A (x_1, y_1), C, B $(x_2 y_2)$

$$C\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

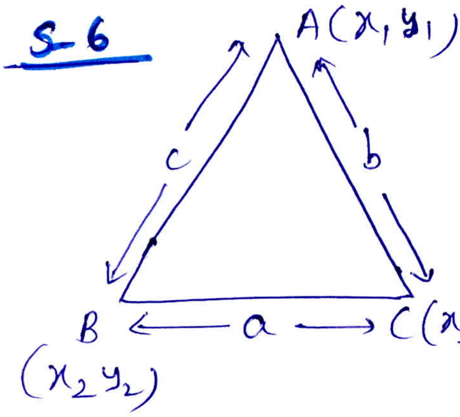
External division

$\leftarrow m \rightarrow$

A (x_1, y_1), B $(x_2 y_2)$ $\leftarrow n \rightarrow$ C

$$C\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

S-6



Centroid $(G) \Rightarrow \left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$ ②

Incentre $(I) \Rightarrow \left(\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

Circumcentre (O)

$$\Rightarrow \left(\frac{x_1\sin 2A+x_2\sin 2B+x_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}, \frac{y_1\sin 2A+y_2\sin 2B+y_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}\right)$$

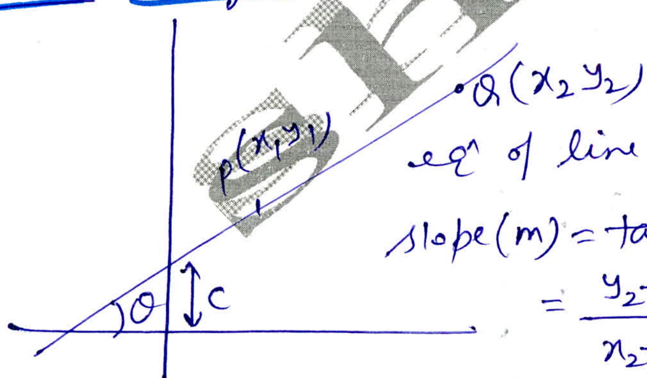
Orthocentre $(H) \Rightarrow \left(\dfrac{x_1\tan A+x_2\tan B+x_3\tan C}{\tan A+\tan B+\tan C}, \dfrac{y_1\tan A+y_2\tan B+y_3\tan C}{\tan A+\tan B+\tan C}\right)$

Note 1) O, G and H of an acute ΔABC are collinear, G divides OH in the ratio 1:2

$OG : GH = 1:2$ $\quad O \underset{1}{—} G \underset{2}{———} H$

2) In an isosceles triangle, G, H, I and O lie on the same line. In an equilateral triangle, all these four points coincide.

S-7 Straight line

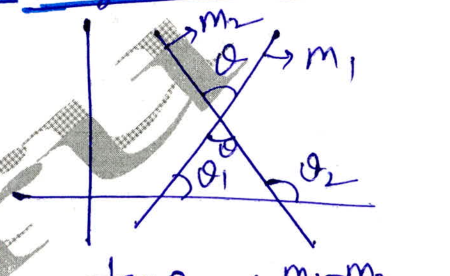


eqⁿ of line : $y = mx + c$

c : y intercept
m : slope of line

slope$(m) = \tan\theta = \dfrac{y_2-y_1}{x_2-x_1}$

S-8 Angle b/w two lines



$\tan\theta = \pm \dfrac{m_1-m_2}{1+m_1m_2}$

$m_1 = \tan\theta_1$

$m_2 = \tan\theta_2$

S-9

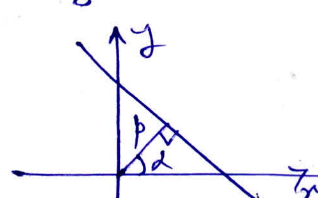
if $m_1 = m_2 \Rightarrow$ lines are parallel to each other

if $m_1m_2 = -1 \Rightarrow$ lines are $\perp$ to each other.

S-10 Eqⁿs of line

1) parallel to x axis : $y = b$
2) parallel to y axis : $x = a$
3) Slope Intercept form: $y = mx + c$
4) Point Slope form : $y - y_1 = m(x - x_1)$
5.) Two point form: $y - y_1 = \dfrac{y_2-y_1}{x_2-x_1}(x - x_1)$
6) Intercept form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$
7) Normal form

$x\cos\alpha + y\sin\alpha = p$



S-11 Condition for two lines

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

$\Rightarrow$ (i) Coincident if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$, (ii) parallel if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

(iii) Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

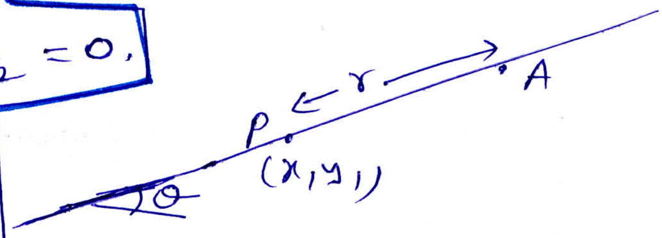
(iv) perpendicular, if $a_1a_2 + b_1b_2 = 0$.

S-12 $ax+by+c=0$ — (X)

eqⁿ of line ∥ to (X) $\quad ax+by+\lambda=0$

eqⁿ of line ⊥ to (X) $\quad bx-ay+\mu=0$

S-13 Parametric form



$$\Rightarrow \frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$$\Rightarrow (x = x_1 + r\cos\theta,\ y = y_1 + r\sin\theta)$$

S-14 Concurrency of three lines

$a_1x+b_1y+c_1=0$

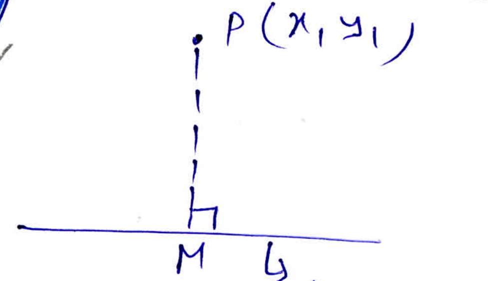
$a_2x+b_2y+c_2=0$

$a_3x+b_3y+c_3=0$

three lines are concurrent

$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

S-15 Distance of a point from a line



$$|PM| = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

S-16

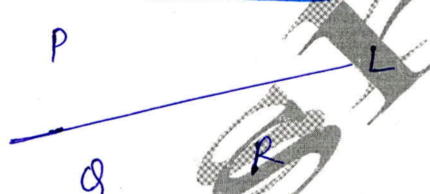
$a_1x+b_1y+c_1=0$

$$|PN| = \frac{|c_1-c_2|}{\sqrt{a_1^2+b_1^2}}$$

P ↕ N parallel lines

$a_1x+b_1y+c_2=0$

S-17 Position of points relative to a line



if $L(P) \times L(Q) < 0 \Rightarrow L(P)$ & $L(Q)$ are of opposite sign

$\Rightarrow$ P & Q are on opposite side of line L.

if $L(Q) \times L(R) > 0 \Rightarrow L(Q)$ & $L(R)$ are of same sign

$\Rightarrow$ Q & R are on same side of line L.

S-18 Eqⁿ of bisectors of the angles b/w the lines

$a_1x+b_1y+c_1=0$ — (i)

$a_2x+b_2y+c_2=0$ — (ii)

Eqⁿ of angle bisectors of two lines (i) & (ii)

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = +\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \quad \text{— (X)}$$

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = -\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \quad \text{— (Y)}$$

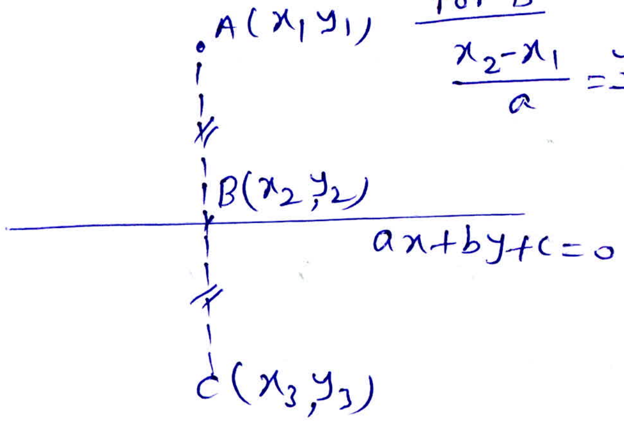
Make c_1, c_2 +ve then

(X) → eqⁿ containing Origin

(Y) → eqⁿ Not containing Origin

Condition	Acute bisector	Obtuse bisector	
$a_1a_2+b_1b_2>0$	(Y)	(X)	Origin lies in obtuse angle
$a_1a_2+b_1b_2<0$	(X)	(Y)	Origin lies in acute angle

S-19 Foot of perpendicular and Image of a Point in a Line



For B

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=-\frac{(ax_1+by_1+c)}{a^2+b^2}$$

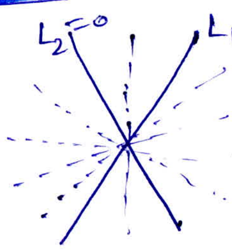
For C

$$\frac{x_3-x_1}{a}=\frac{y_3-y_1}{b}=-\frac{2(ax_1+by_1+c)}{a^2+b^2}$$

S-21 separate lines

$a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ then their combined eq^n be $(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=0$

S-20 Family of Straight lines



family of lines passing thru the pt of intersection of two lines is $L_1+\lambda L_2=0$ $(\lambda\in R)$

S-22 Pair of st. lines

$ax^2+2hxy+by^2$ (homogeneous 2^nd degree eq^n represents two lines passing thru origin)

$y=m_1x$ & $y=m_2x$

such that $m_1+m_2=-\frac{2h}{b}$

$m_1m_2=\frac{a}{b}$

S-24 Bisectors of angle b/w the lines represented by $ax^2+2hxy+by^2=0$ is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

S-25 General 2^nd degree eq^n

$ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two lines if

$\Delta(abc+2fgh-af^2-bg^2-ch^2)=0$

and angle b/w two lines, $\tan\theta=\pm\frac{2\sqrt{h^2-ab}}{a+b}$

and pt of intersection of the two lines is $\left(\frac{bg-hf}{h^2-ab},\frac{af-gh}{h^2-ab}\right)$

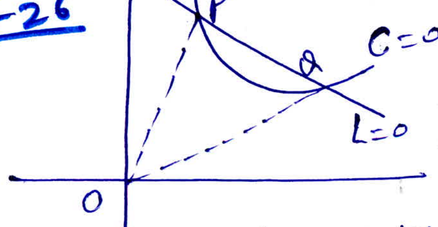
S-23 Angle b/w two lines represented by $ax^2+2hxy+by^2=0$

is $\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

if $a+b=0 \Rightarrow$ two lines are $\perp$ to each other

if $h^2=ab \Rightarrow$ two lines are parallel ie coincident.

S-26

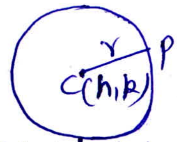


Curve and a line intersect at two points.

Combined eq^n of OP & OQ is obtained by homogenizing the eq^n of curve with the help of eq^n of line.

CIRCLE

C-1



Centre (h,k)
radius r
$(x-h)^2+(y-k)^2=r^2$
if centre $(0,0)$
$\Rightarrow x^2+y^2=r^2$

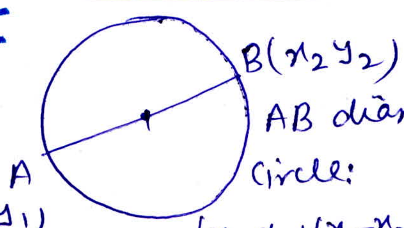
C-2 general eqⁿ
$x^2+y^2+2gx+2fy+c=0$
centre $(-g,-f)$
radius $=\sqrt{g^2+f^2-c}$

C-4 Two lines $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ cut the coordinate axes in four concyclic pts if
$m_1m_2=1 \Rightarrow a_1a_2=b_1b_2$

C-3 General 2^{nd} degree eqⁿ
$ax^2+2hxy+by^2+2gx+2fy+c=0$

- Pair of st lines $(\Delta=0)$
- Circle $(\Delta\neq0,\ a=b,\ h=0)$
- Parabola $(\Delta\neq0,\ h^2=ab)$
- Ellipse $(\Delta\neq0,\ h^2<ab)$
- Hyperbola $(\Delta\neq0,\ h^2>ab)$

C-5



AB diameter
Circle:
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

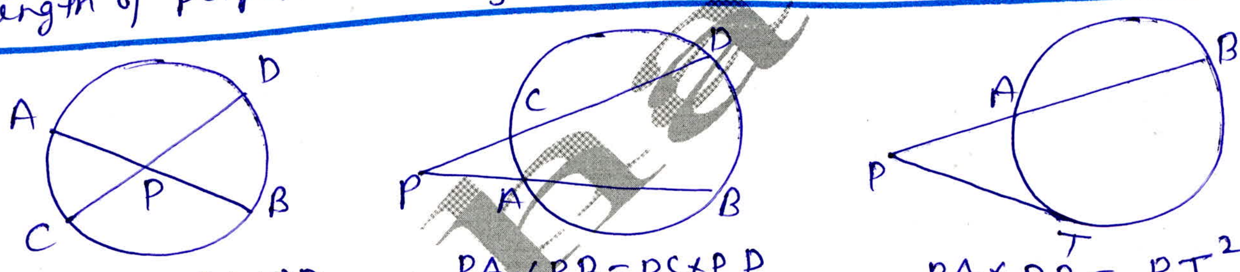
C-6 Parametric pt on $x^2+y^2=r^2$ is $(r\cos\theta, r\sin\theta)$ $(0\leq\theta\leq2\pi)$

C-7 Intercepts made on Axes by $x^2+y^2+2gx+2fy+c=0$
x intercept $=2\sqrt{g^2-c}$
y intercept $=2\sqrt{f^2-c}$
if circle touches both the axes
$\Rightarrow g^2=f^2=c$

C-8 Position of a point wrt a circle
$x^2+y^2+2gx+2fy+c=0 \Rightarrow S=0$
P pt lies outside, on or inside the circle accordingly as $S(P)>,=,<0$

C-9 a line intersects, touches or does not intersect the circle if radius of circle is greater than, equal to or less than the length of perpendicular from centre of the circle to the line.

C-10



$\Rightarrow PA\times PB=PC\times PD$ $\qquad PA\times PB=PC\times PD$ $\qquad PA\times PB=PT^2$

C-11 If two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ meet the axes in four distinct concyclic pts then $a_1a_2=b_1b_2$ and also the eqⁿ of circle passing thru those concyclic pts is
$(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)-(a_1b_2+a_2b_1)xy=0$

C-12 The eqⁿ of the circumcircle of Δ formed by the line $ax+by+c=0$ with the coordinate axes is
$ab(x^2+y^2)+c(bx+ay)=0$

C-13 eq^n of tangent to circle $x^2+y^2+2gx+2fy+c=0$ at pt (x_1, y_1) is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

Note to write eq^n of tangent to any curve at pt (x_1, y_1) make the following changes in the eq^n of curve

$x^2 \to xx_1$

$y^2 \to yy_1$

$2x \to x+x_1$

$2y \to y+y_1$

$2xy \to xy_1+yx_1$

keep the const as such

C-14 Circle $x^2+y^2=a^2$

tangent at (x_1, y_1): $xx_1+yy_1-a^2=0$

tangent at $(a\cos\theta, a\sin\theta)$: $x\cos\theta+y\sin\theta-a=0$

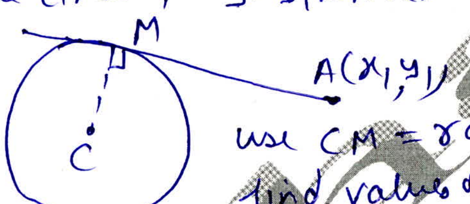
tangent in slope (m) form

$y=mx \pm a\sqrt{1+m^2}$

hence a line $y=mx+c$ is a tangent to $x^2+y^2=a^2$ if $c^2=a^2(1+m^2)$

C-15 eq^n of tangent to the circle $x^2+y^2+2gx+2fy+c=0$ in terms of slope is $y+f=m(x+g) \pm \sqrt{g^2+f^2-c}\sqrt{1+m^2}$

C-16 tangent from a point outside the circle, $y-y_1=m(x-x_1)$



use CM = radius to find value of m.

C-17 To find the pt of contact of a given tangent with the circle, write the eq^n of tangent at (x_1, y_1) to the circle, then compare this eq^n with the given tangent eq^n to find x_1 & y_1.

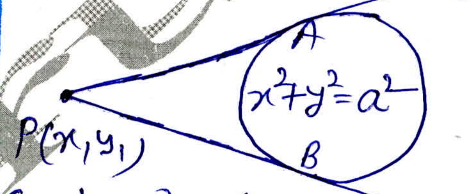
C-18 length of tangent from a Point to a Circle

$\to x^2+y^2+2gx+2fy+c=0$

$|PT| = \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

$= \sqrt{S_1}$

C-19 Pair of tangents



Combined eq^n of PA & PB is given by

$SS_1 = T^2$

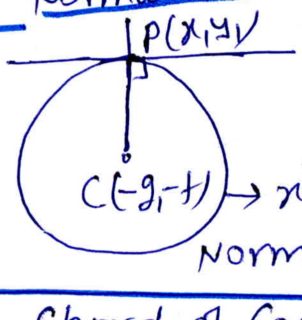
where

$S: x^2+y^2-a^2$

$S_1: x_1^2+y_1^2-a^2$

$T: xx_1+yy_1-a^2$

C-20 Normal to a Circle



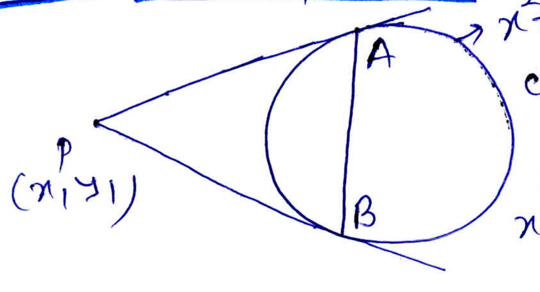
normal CP:

$(y-y_1) = \frac{y_1+f}{x_1+g}(x-x_1)$

$\to x^2+y^2+2gx+2fy+c=0$

Normal to a circle always passes thru centre of Circle

C-21 Chord of Contact



$\to x^2+y^2=a^2$

chord of Contact

AB: $T=0$

ie

$xx_1+yy_1-a^2=0$

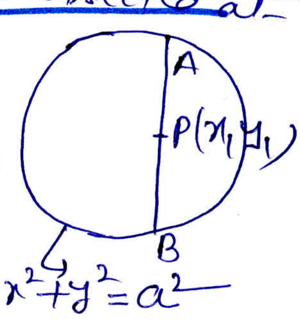
C-22 Eq^n of chord bisected at (x_1, y_1)

AB: $T=S_1$

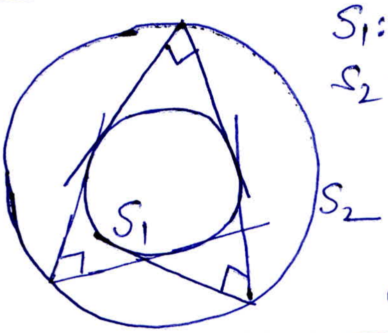
where

$T: xx_1+yy_1-a^2=0$

$S_1: x_1^2+y_1^2-a^2=0$



C-23 Director Circle



$S_1: x^2+y^2=a^2$

S_2: DC of S_1

$x^2+y^2=2a^2$

Radius of DC is $\sqrt{2}$ times the radius of origin circle.

C-24 Angle of Intersection of two Circles

$S: x^2+y^2+2g_1x+2f_1y+c_1=0$

$S': x^2+y^2+2g_2x+2f_2y+c_2=0$

if angle b/w them is θ

then $\cos\theta = \left|\dfrac{r_1^2+r_2^2-d^2}{2r_1r_2}\right|$

where r_1, r_2 are radii of two circles and 'd' is the distance b/w their centres

if $\theta = 90°$, then the circles are said to be orthogonal circles, Condition for orthogonality is

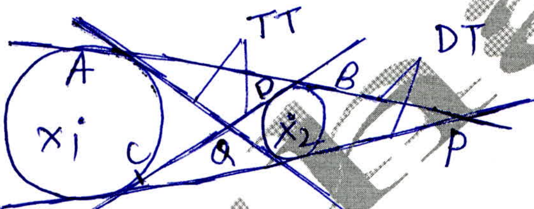
$r_1^2+r_2^2-d^2=0 \Rightarrow 2g_1g_2+2f_1f_2=c_1+c_2$

C-25 Intersection of two Circles

$x^2+y^2+2g_1x+2f_1y+c_1=0$, centre $X_1(-g_1,-f_1)$ radius r_1

$x^2+y^2+2g_2x+2f_2y+c_2=0$, centre $X_2(-g_2,-f_2)$ radius r_2

(i) $|X_1X_2| > r_1+r_2$



2 Direct Common tangents (DT)

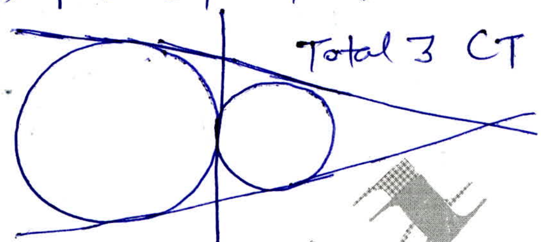
2 transverse common tangents (TT)

Total 4 Common tangents (CT)

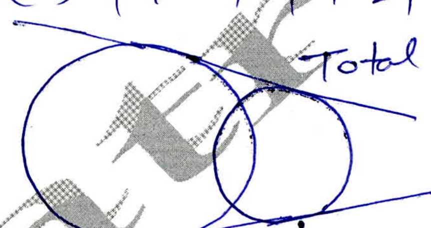
P divides X_1, X_2 externally in the ratio $r_1 : r_2$

Q divides X_1, X_2 internally in the ratio $r_1 : r_2$.

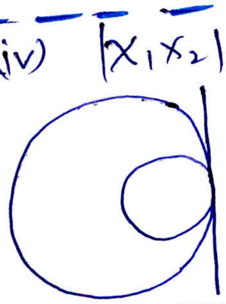
(ii) $|X_1X_2| = r_1+r_2$



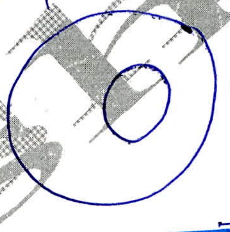
(iii) $|r_1-r_2| < |X_1X_2| < |r_1+r_2|$



(iv) $|X_1X_2| = |r_1-r_2|$



(v) $|X_1X_2| < |r_1-r_2|$



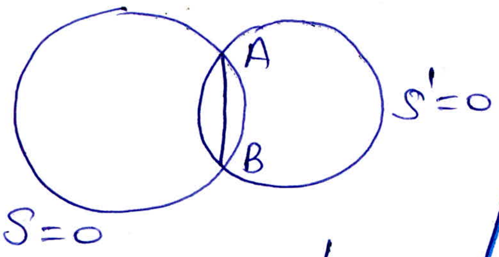
C-26 Length of Common tangents

length of direct common tangent $|AB| = \sqrt{d^2-(r_1-r_2)^2}$

length of transverse common tangent $|CD| = \sqrt{d^2-(r_1+r_2)^2}$

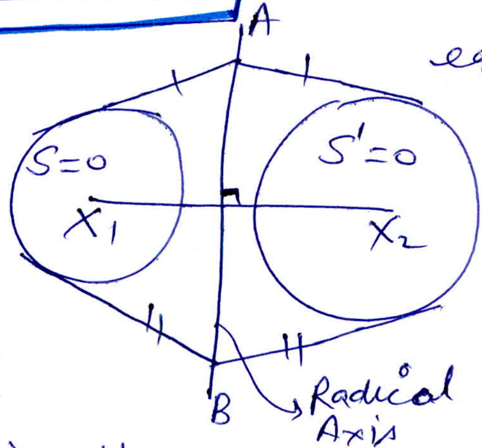
where d is the distance b/w centres of two Circles

C-27 Common Chord of two Circles



eq^n of AB: $S - S' = 0$

C-28 Radical Axis



eq^n of Radical Axis

$S - S' = 0$

Properties of Radical Axis

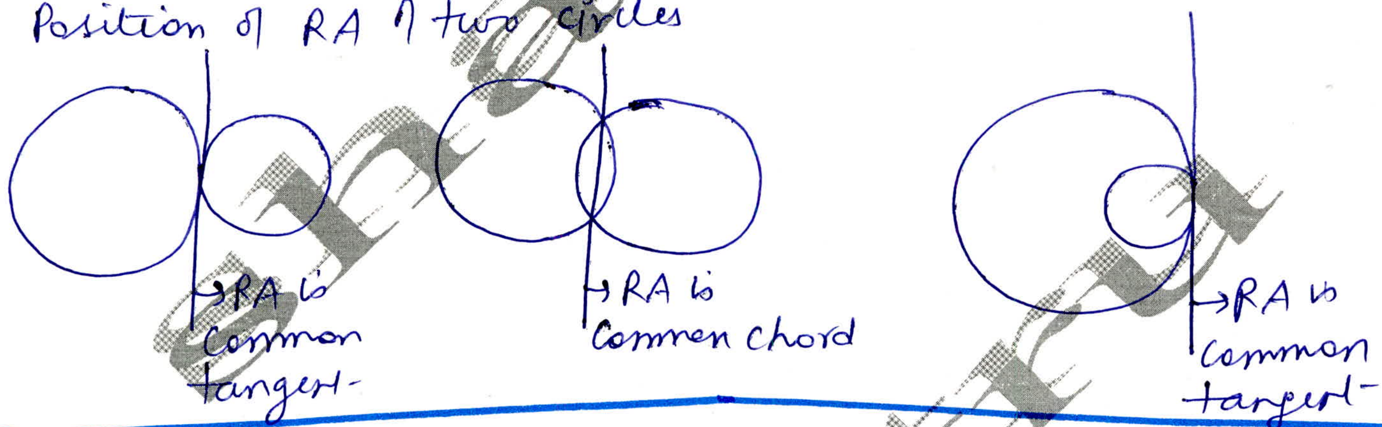
(i) RA is ⊥ to the line joining the centres of the given circles

(ii) RA bisects the common tangents of two circles

(iii) RA need not always pass thru the mid point of the line joining the centres of the two circles

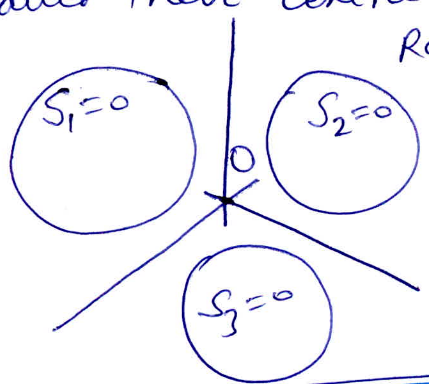
(iv) If two circles cut a third circle orthogonally, then the RA of the two circles will pass thru the centre of the third circle.

(v) Position of RA of two circles



C-29 Radical Centre

RAs of three circles, taken in pairs, meet in a point, which is called their centre.



Radical centre O is pt of intersection of

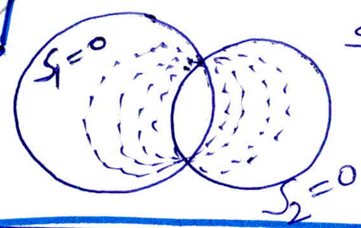
$S_1 - S_2 = 0$

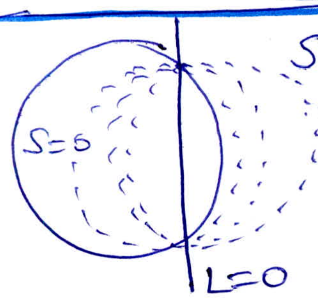
$S_2 - S_3 = 0$

$S_3 - S_1 = 0$

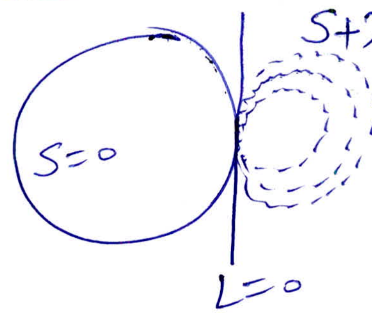
C-30 Family of Circles

$S_1 + \lambda S_2 = 0$

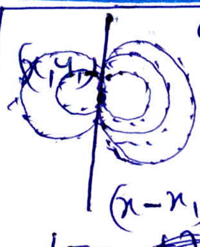




$S + \lambda L = 0$



$S + \lambda L = 0$

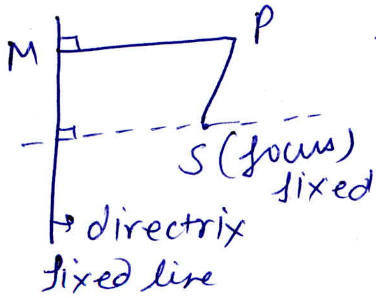


Circles touch a line at (x_1, y_1).

$(x - x_1)^2 + (y - y_1)^2 + \lambda L = 0$

PARABOLA

P-1 Conic Section as a Locus of a Point



$\frac{PS}{PM} = e$ (eccentricity)

$e = 0 \Rightarrow$ Circle
$e = 1 \Rightarrow$ Parabola
$e < 1 \Rightarrow$ Ellipse
$e > 1 \Rightarrow$ Hyperbola
$e = \infty \Rightarrow$ Pair of st. lines.

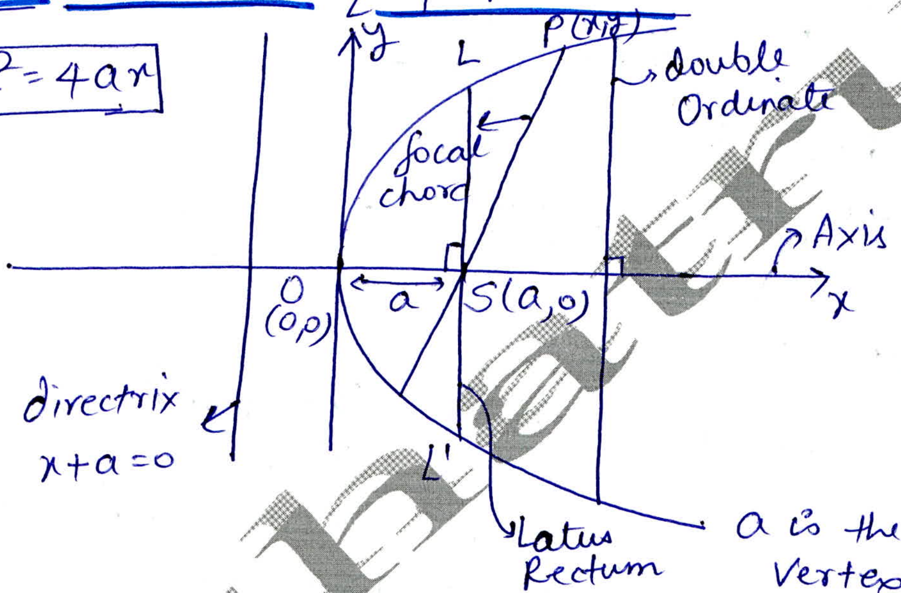
Eqⁿ of Conic Section

Focus (α, β), Directrix $(ax+by+c=0)$

$PS = e\,PM \Rightarrow (x-\alpha)^2+(y-\beta)^2 = e^2\left(\frac{ax+by+c}{a^2+b^2}\right)$

P-2 Standard Eq of Parabola

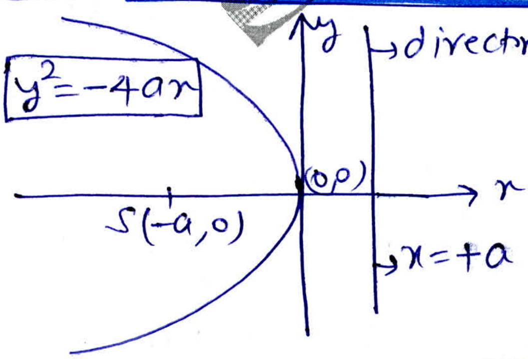
$\boxed{y^2 = 4ax}$

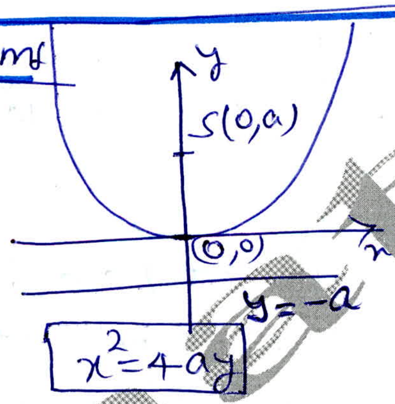


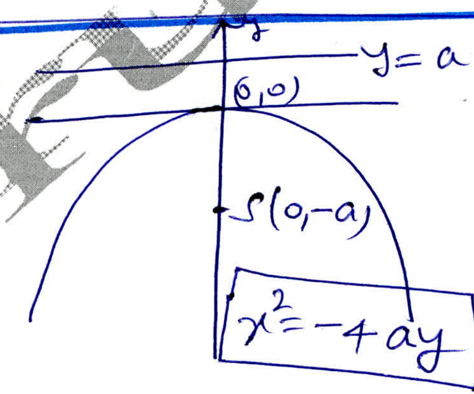
Vertex $(0,0)$
Tangent at vertex $x=0$
Axis : $y=0$
End pts of LR: $(a, \pm 2a)$
length of LR : $4a$
focal distance $SP = x+a$
eqⁿ of LR: $x=a$
Parametric form
$x = at^2$, $y = 2at$
a is the distance b/w focus and Vertex.

P-3 Other Standard forms

$\boxed{y^2 = -4ax}$







P-4

Focus $(q,0)$, Vertex $(p,0)$
eqⁿ of Parabola
$y^2 = 4(q-p)(x-p)$ $(p<q)$
or
$y^2 = -4(p-q)(x-p)$ $(q<p)$

Focus $(0,q)$ Vertex $(0,p)$
eqⁿ of Parabola
$x^2 = 4(q-p)(y-p)$ $(q>p)$
or
$x^2 = -4(p-q)(y-p)$ $(p>q)$

P-5 Eqⁿ of Parabola when Vertex is (h,k) and Axis is Parallel to Coordinate Axes

$(y-k)^2 = 4a(x-h)$ and $(x-h)^2 = 4a(y-k)$

Axis is $\parallel$ to x-axis

axis is $\parallel$ to y axis

P-6 Position of a point wrt a Parabola $y^2=4ax$

$$\underbrace{y^2-4ax}_{S}=0$$

- P point lies outside, on or inside the Parabola if
- $S(P) > 0, =, < 0$

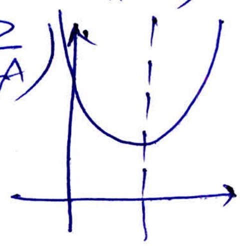
P-7 Parabolic Curve

$y = Ax^2+Bx+C$

$\Rightarrow \left(x+\frac{B}{2A}\right)^2 = \frac{1}{A}\left(y+\frac{B^2-4AC}{4A}\right)$

Vertex $\left(-\frac{B}{2A}, -\frac{D}{4A}\right)$

Length of $LR = \frac{1}{|A|}$

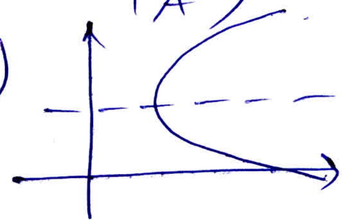


$x = Ay^2+By+C$

$\Rightarrow \left(y+\frac{B}{2A}\right)^2 = \frac{1}{A}\left(x+\frac{B^2-4AC}{4A}\right)$

Vertex $\left(-\frac{D}{4A}, -\frac{B}{2A}\right)$

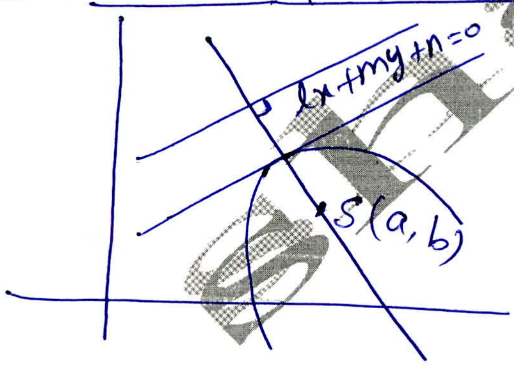
Length of $LR = \frac{1}{|A|}$



P-8 Parametric form of parabola $(y-k)^2 = 4a(x-h)$ is

$x = h+at^2, \quad y = k+2at$

P-9 General eq^n of a Parabola



$SP = PM \Rightarrow (x-a)^2+(y-b)^2 = \frac{(lx+my+n)^2}{l^2+m^2}$

$\Rightarrow m^2x^2+l^2y^2-2lmxy + x\text{ term} + y\text{ term} + \text{const} = 0$

$\Rightarrow (mx-ly)^2+2gx+2fy+c=0$

Note: second degree terms in the general eq^n of a parabola forms a perfect square.

P-10 Properties of focal chord

(i) If the chord joining $P\equiv(at_1^2, 2at_1)$ and $Q\equiv(at_2^2, 2at_2)$ is the focal chord then $t_1t_2=-1 \Rightarrow P(at^2, 2at), Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

(ii) If point P is $(at^2, 2at)$, then length of focal chord PQ is $a\left(t+\frac{1}{t}\right)^2$

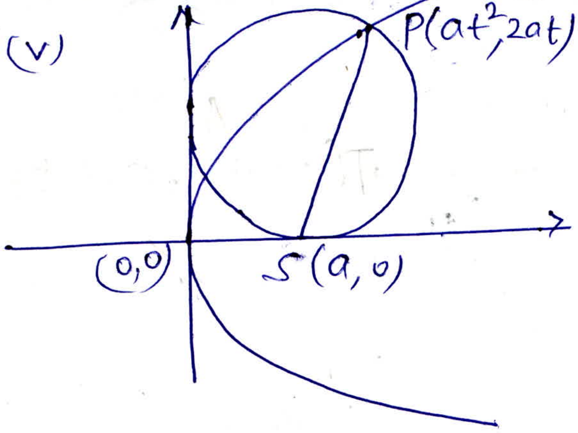
(iii) The length of focal chord which makes an angle θ with the direction of x-axis is $4a\,\text{cosec}^2\theta$.

(iv) Semi-Latus Rectum is harmonic mean of SP and SQ, where P & Q are extremities of focal chord. (S focus)

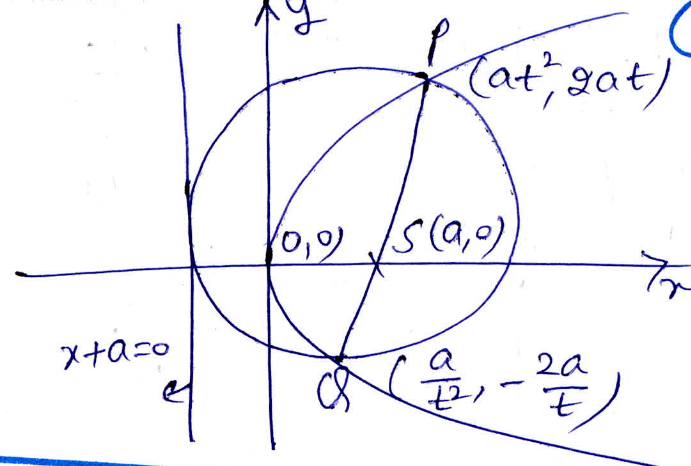
(v) Circle described on the focal length as diameter touches tangent at vertex

(vi) Circle described on the focal chord as diameter touches directrix.

(v)



(vi)



P-11 Eq^n of tangent

Eq^n of tangent at pt (x_1, y_1) to Parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$

Parametric form : $ty = x + at^2$ at $(at^2, 2at)$

slope form(m) : $y = mx + \frac{a}{m}$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

For $(y-k)^2 = 4a(x-h)$: eq^n of tangent in slope form(m)

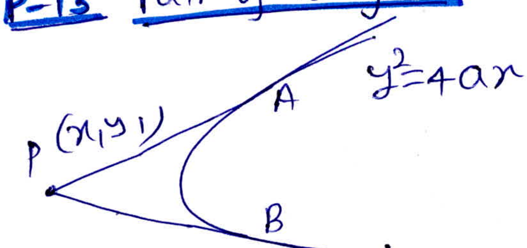
: $(y-k) = m(x-h) + \frac{a}{m}$

line $y = mx + c$ touches $y^2 = 4ax$ then $c = \frac{a}{m}$

P-12 Note

Eq^n of Parabola	Tangent at t	Tangent in slope form(m)
$y^2 = 4ax$	$ty = x + at^2$ at $(at^2, 2at)$	$y = mx + \frac{a}{m}$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
$y^2 = -4ax$	$ty = -x + at^2$ at $(-at^2, 2at)$	$y = mx - \frac{a}{m}$ at $\left(-\frac{a}{m^2}, -\frac{2a}{m}\right)$
$x^2 = 4ay$	$tx = y + at^2$ at $(2at, at^2)$	$y = mx - am^2$ at $(2am, am^2)$
$x^2 = -4ay$	$tx = -y + at^2$ at $(2at, -t^2)$	$y = mx + am^2$ at $(-2am, -am^2)$

P-13 Pair of tangents



Combined eq^n of PA & PB is

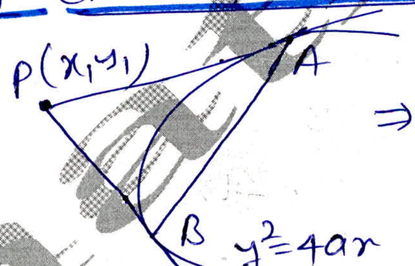
$SS_1 = T^2$

where $S: y^2 - 4ax$

$S_1 : y_1^2 - 4ax_1$

$T : yy_1 - 2a(x + x_1) = 0$

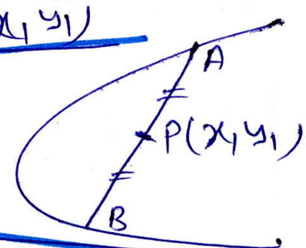
P-14 Chord of Contact



AB: $T = 0$

$\Rightarrow yy_1 - 2a(x + x_1) = 0$

P-15 Eq^n of chord whose mid pt is (x_1, y_1)



AB: $T = S_1$

where

$T : yy_1 - 2a(x + x_1) = 0$

$S_1 : y_1^2 - 4ax_1$

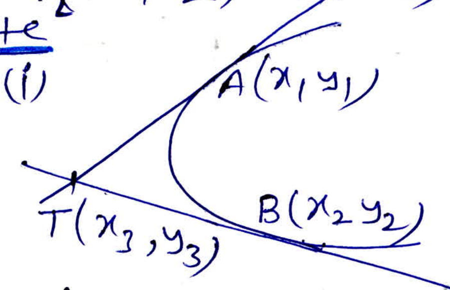
P-16 Parabola has No Centre, but circle, Ellipse, Hyperbola have centre.

P-17 Properties of tangents

(i) Points of intersection of Tangents at any two pts $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$ on the Parabola $y^2=4ax$ is $T\{at_1t_2, a(t_1+t_2)\}$

Note

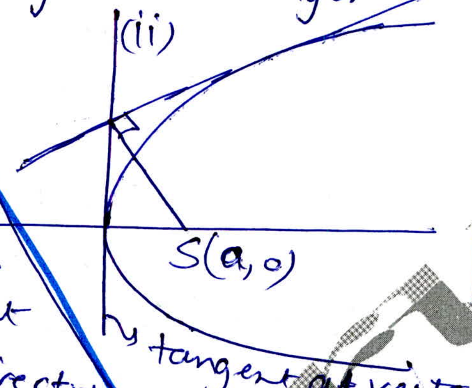
(i)



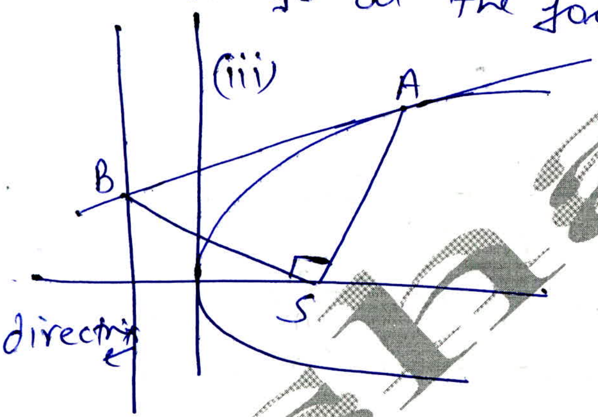
$x_3 = GM(x_1, x_2) = \sqrt{x_1x_2}$

$y_3 = AM(y_1, y_2) = \frac{y_1+y_2}{2}$

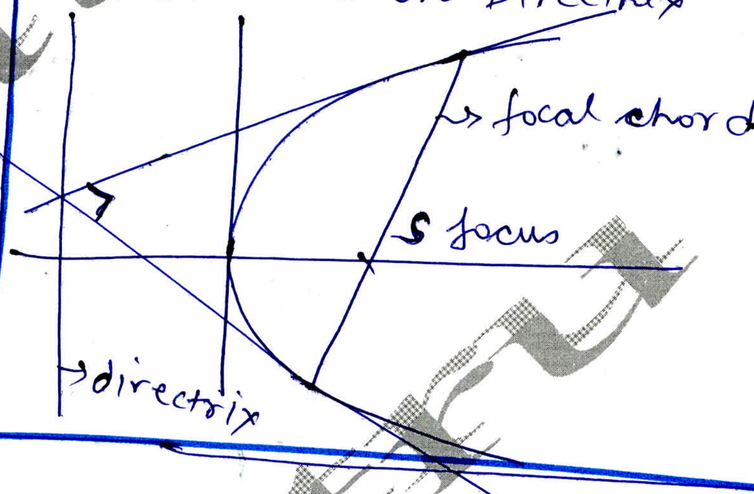
(ii) Locus of foot of perpendicular from focus upon any tangent is tangent at vertex



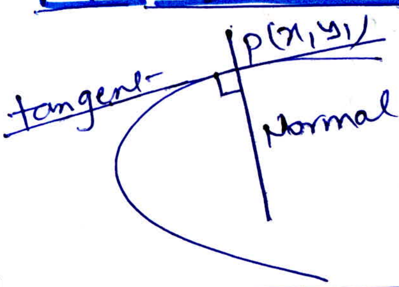
(iii) Length of tangent between the point of contact and the point where it meets the directrix subtends 90° at the focus



(iv) Tangents at Extremities of focal chord are perpendicular and Intersect on Directrix



P-18 Equation of Normal



eq^n of Normal to $y^2=4ax$ at (x_1,y_1) is

$y-y_1 = -\frac{y_1}{2a}(x-x_1)$ (Point form)

$y = -tx+2at+at^3$ (Parametric form)

$y = mx-2am-am^3$ (Slope form)

Parabola	Normal (Parametric)	Normal (Slope form)
$y^2=4ax$	$y=-tx+2at+at^3$ at $(at^2, 2at)$	$y=mx-2am-am^3$ at $(am^2, -2am)$
$y^2=-4ax$	$y=tx+2at+at^3$ at $(-at^2, 2at)$	$y=mx+2am+am^3$ at $(-am^2, 2am)$
$x^2=4ay$	$x=-ty+2at+at^3$ at $(2at, at^2)$	$y=mx+2a+\frac{a}{m^2}$ at $\left(-\frac{2a}{m}, \frac{a}{m^2}\right)$
$x^2=-4ay$	$x=ty+2at+at^3$ at $(2at, -at^2)$	$y=mx-2a-\frac{a}{m^2}$ at $\left(\frac{2a}{m}, -\frac{a}{m^2}\right)$

P-19 Properties of Normal

(i) Normal other than axis of Parabola never passes thru focus.

(ii) Point of intersection of normals at $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ is $[2a+a(t_1^2+t_2^2+t_1t_2), -at_1t_2(t_1+t_2)]$

(iii) Normal at point $P(t_1)$ meets the curve again at pt $Q(t_2)$ such that $t_2 = -t_1 - \frac{2}{t_1}$

P-20 Co-Normal Points

$y = mx - 2am - am^3 \Rightarrow am^3 + m(2a-h) + k = 0$ (cubic in m)

$\Rightarrow m_1+m_2+m_3 = 0$ & $m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$

Hence in total, we have maximum three normals. Points in which the three normals from (h,k) meet the Parabola are called Co-Normal Points.

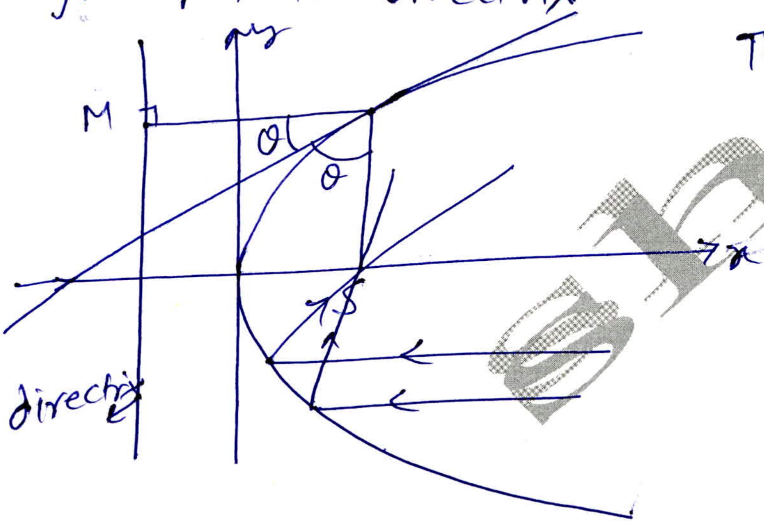
Note (i) The algebraic sum of ordinates of the feet of three normals drawn to a parabola from a given point is 0.

(ii) Centroid of the triangle formed by the feet of the three normals lies on the axis of the parabola.

(iii) If three normals drawn to any parabola $y^2 = 4ax$ from a given point (h,k) be real, then $h > 2a$.

P-21 Reflection Property of Parabola

The tangent at any point P to a parabola bisects the angle between the focal chord through P and the perpendicular from P to the directrix.



Thus, if any light ray is sent along a line parallel to the axis of the parabola then the reflected ray passes through the focus, as the normal bisects the angle between the incident ray and reflected ray.

P-22 Tangents are drawn from the point (x_1, y_1) to the parabola $y^2 = 4ax$, the length of their chord of contact $= \frac{1}{|a|}\sqrt{(y_1^2 - 4ax_1)(y_1^2 + 4a^2)}$

P-23 Area of the triangle formed by the tangents drawn from (x_1, y_1) to $y^2 = 4ax$ and their chord of contact is

$$\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}.$$

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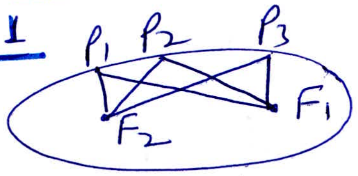
ELLIPSE

E-1



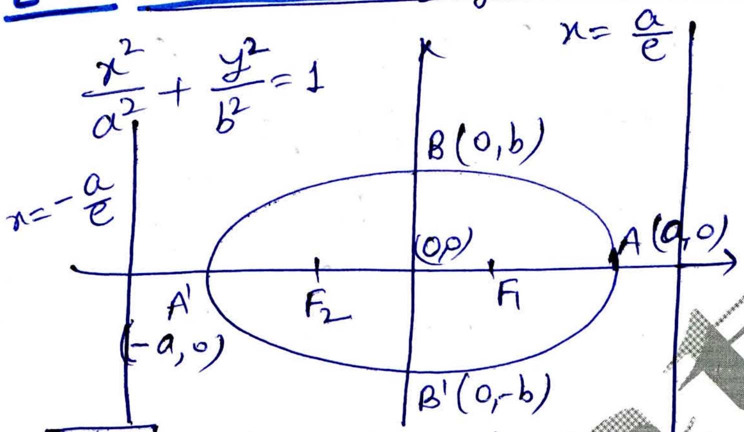
An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. The two fixed points are called the foci of the ellipse

Foci F_1 and F_2

$\Rightarrow P_1F_1 + P_1F_2 = P_2F_1 + P_2F_2 = P_3F_1 + P_3F_2$

E-2 Standard Eq^n of an Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$\boxed{a>b}$, $AA' \Rightarrow$ major axis $= 2a$

$BB' \Rightarrow$ minor axis $= 2b$

Foci $\Rightarrow (\pm ae, 0)$

Directrix $\Rightarrow x = \pm \frac{a}{e}$

$PF_1 + PF_2 = 2a$

$b^2 = a^2(1-e^2)$

Vertices $(\pm a, 0)$

Length of Latus Rectum $= \frac{2b^2}{a}$

Ends of LR: $\left(\pm ae, \pm \frac{b^2}{a}\right)$

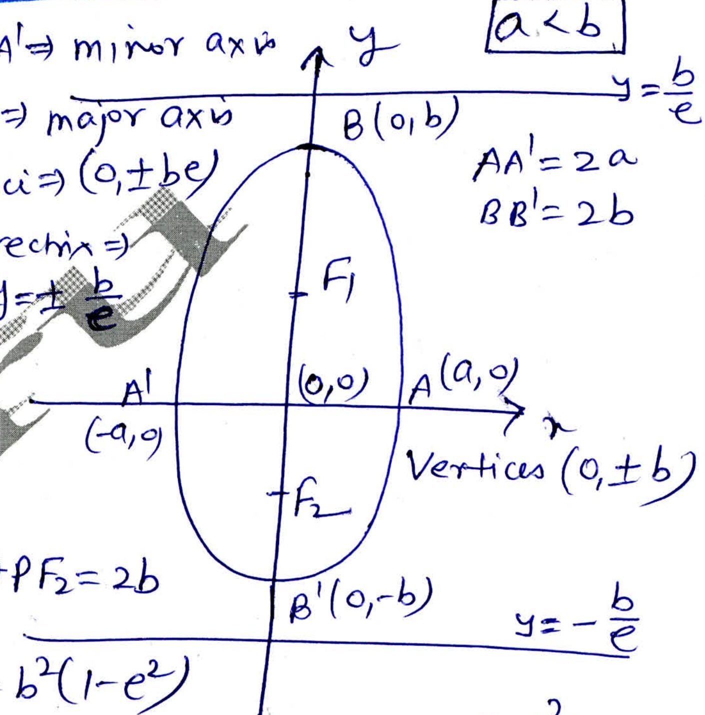
$\boxed{a<b}$

$AA' \Rightarrow$ minor axis

$BB' \Rightarrow$ major axis

Foci $\Rightarrow (0, \pm be)$

Directrix $\Rightarrow$ $y = \pm \frac{b}{e}$



$AA' = 2a$

$BB' = 2b$

Vertices $(0, \pm b)$

$PF_1 + PF_2 = 2b$

$a^2 = b^2(1-e^2)$

length of Latus Rectum $= \frac{2a^2}{b}$

Ends of LR: $\left(\pm \frac{a^2}{b}, \pm be\right)$

$$e = \frac{\text{distance from centre to focus}}{\text{distance from centre to vertex}}$$

E-3 Two ellipses are said to be similar if they have the same value of eccentricity.

E-4 Eq^n of an Ellipse whose axes are parallel to Coordinate Axes and Centre is (h,k)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (a>b)$$

Foci: $(h \pm ae, k)$

Directrix: $x = h \pm \frac{a}{e}$

E-5 length of Latus Rectum $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$ $(a>b)$.

$LR = \frac{2b^2}{a}$

$= \frac{4b^2}{2a} = \frac{(\text{minor axis})^2}{(\text{major axis})}$

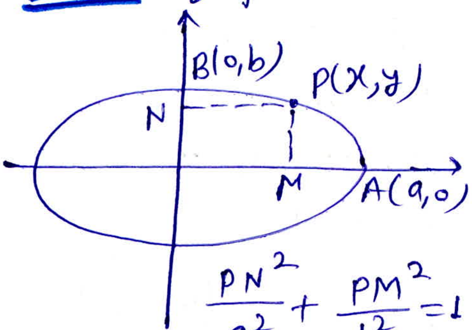
$= 2e\left(\frac{a}{e} - ae\right)$

$= 2e$(distance b/w focus and corresponding foot of directrix)

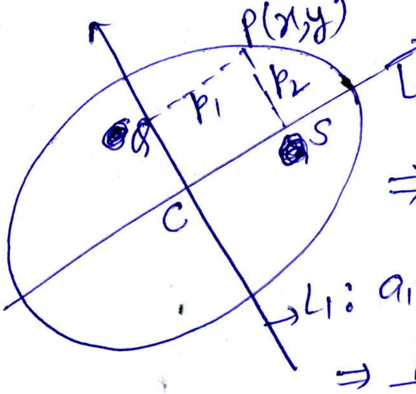
E-6 Position of a point wrt an Ellipse

Point $P(h,k)$ will lie outside, on or inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ according to $\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1 >, =, < 0$.

E-7 Eq^n of an Ellipse referred to two perpendicular lines



$\Rightarrow$



$$\frac{PN^2}{a^2} + \frac{PM^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$L_2 : b_1x - a_1y + c_2 = 0$

$$\Rightarrow \frac{PQ^2}{a^2} + \frac{PS^2}{b^2} = 1$$

$L_1 : a_1x + b_1y + c_1 = 0$

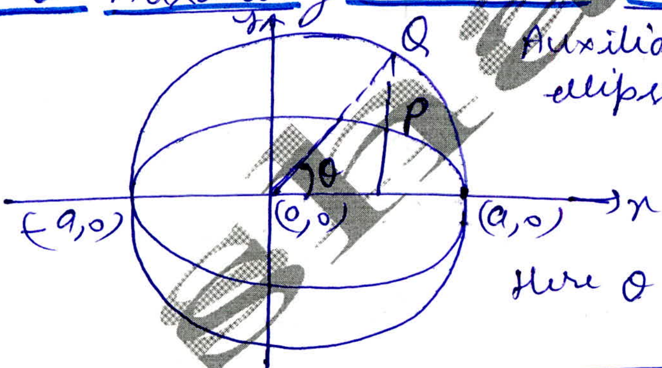
$$\Rightarrow \frac{p_1^2}{a^2} + \frac{p_2^2}{b^2} = 1$$

$$\Rightarrow \frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{a^2} + \frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{a_1^2+b_1^2}}\right)^2}{b^2} = 1$$

Note (i) In the above mentioned ellipse, the centre of the ellipse is the point of intersection of the lines $L_1 = 0$ & $L_2 = 0$

(ii) The major axis lies along $L_2 = 0$ and the minor axis lies along $L_1 = 0$ if $a > b$.

E-8 Auxiliary Circle and Eccentric Angle



Auxiliary circle : $x^2+y^2 = a^2$

ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$Q(a\cos\theta, a\sin\theta)$, $0 \le \theta \le 2\pi$

$P(a\cos\theta, b\sin\theta)$

Here θ is called eccentric angle of point P.

E-9 Some Important Properties of Ellipse

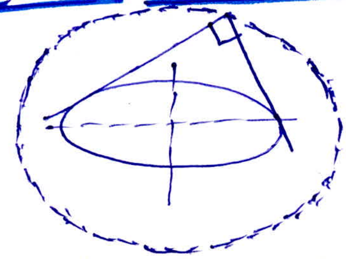
(i) Area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(ii) Ratio of area of any triangle PQR inscribed in ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and that of triangle formed by corresponding points on the auxiliary circle is b/a.

(iii) Semi-Latus Rectum is harmonic mean of segments of focal chord or $\frac{1}{SP} + \frac{1}{SQ} = \frac{2a}{b^2}$ $(a > b)$ (where PQ is focal chord thru focus S)

(iv) Circle described on focal length as diameter always touches auxiliary circle.

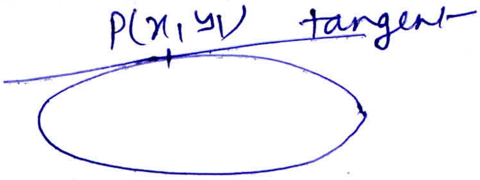
E-10 Director Circle



Director Circle of any ellipse is a circle whose centre is the centre of the ellipse and whose radius is the length of the line joining the ends of major and minor axes.

$x^2+y^2 = a^2+b^2$

E-11 Equation of tangent



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

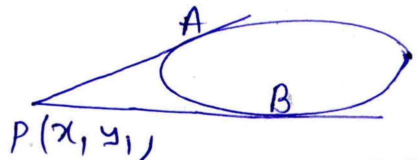
(i) Point form (x_1,y_1) : $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

(ii) Parametric form $(a\cos\theta, b\sin\theta)$: $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

(iii) slope form (m) : $y=mx\pm\sqrt{a^2m^2+b^2}$

Note line $y=mx+c$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $c^2=a^2m^2+b^2$

E-12 Pair of tangents

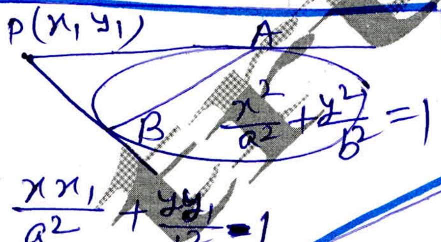


$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, Pair of tangents $SS_1=T^2$, where $S: \frac{x^2}{a^2}+\frac{y^2}{b^2}-1$

$S_1: \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

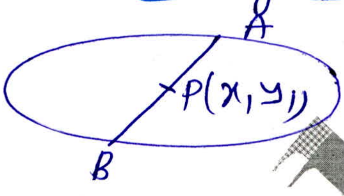
$T: \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$

E-13 Chord of Contact



AB: $T=0$ where $T: \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$

E-14 Eqⁿ of chord of ellipse whose mid point is (x_1,y_1)



AB: $T=S_1$

where $T: \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$

$S_1: \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

E-15 Important Properties related to tangents

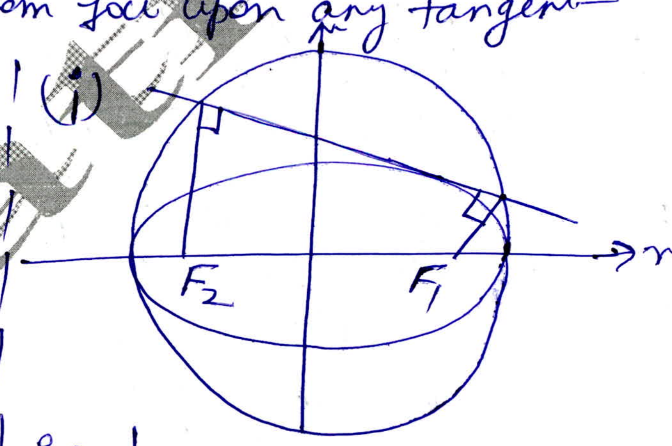
(i) Locus of feet of perpendiculars from foci upon any tangent is an auxiliary circle

(ii) Product of lengths of perpendiculars from foci upon any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is b^2.

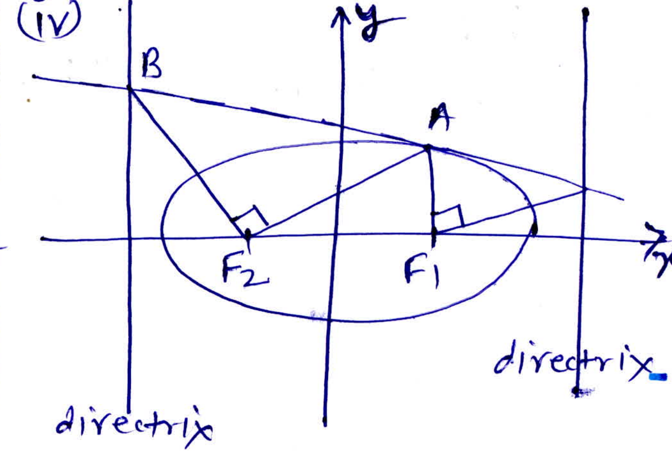
(iii) Tangents at the extremities of Latus Rectum pass through the corresponding foot of directrix on major axis.

(iv) Length of tangent b/w the point of contact and the point where it meets the directrix subtends right angle at the corresponding focus.

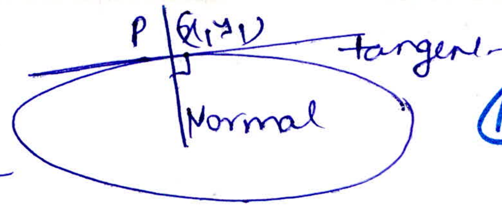
(i)



(iv)



E-16 Equation of Normal



ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(i) Point form (x_1, y_1) : $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

(ii) Parametric form $(a\cos\theta, b\sin\theta)$: $ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2-b^2$

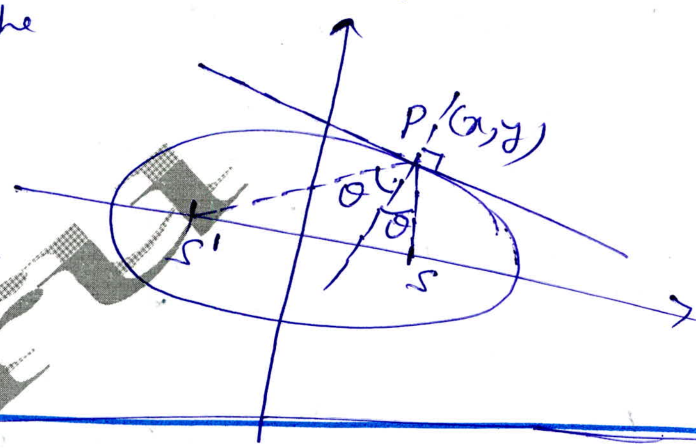
Properties

(i) Normal other than major axis never passes through the focus.

(ii) Normal at the point P bisects the angle SPS'

$SP = a-ex$, $S'P = a+ex$

Thus the incident ray from focus S after reflection by ellipse at point P passes through other focus S'.



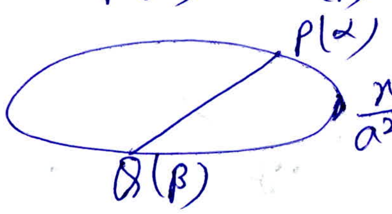
E-17 Co-Normal Points

From any point in the plane maximum four normals can be drawn to ellipse.
Four feet of normals on the ellipse are called Co-Normal points. The condition for their eccentric angles is
$\alpha+\beta+\gamma+\delta = (2n+1)\pi$, $n\in Z$.

E-18 Concyclic points on Ellipse

Let the circle $x^2+y^2+2gx+2fy+c=0$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in four points P, Q, R, S.
Condition: $\alpha+\beta+\gamma+\delta = 2n\pi$, $n\in Z$, where $\alpha, \beta, \gamma, \delta$ are eccentric angles of P, Q, R, S.

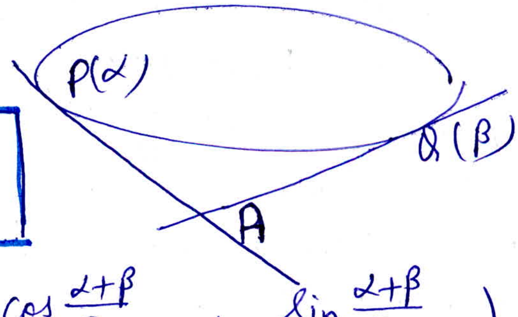
E-19 Eq^n of chord joining pts $P(\alpha)$ & $Q(\beta)$



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

PQ: $\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$

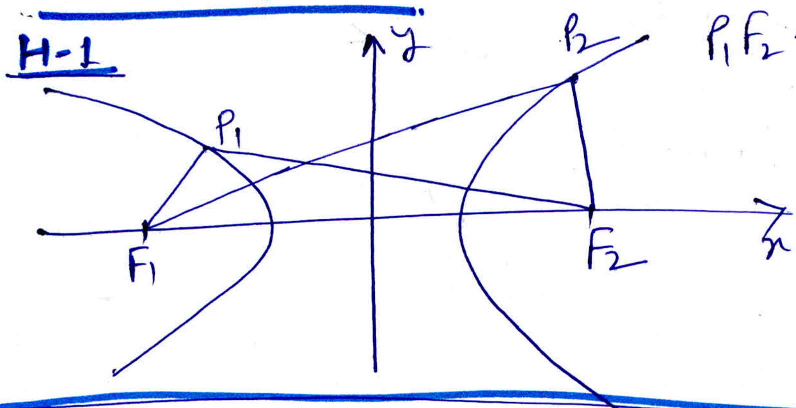
E-20 Point of intersection of tangents at pts $P(\alpha)$ & $Q(\beta)$



$A\left(a\frac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, b\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$

HYPERBOLA

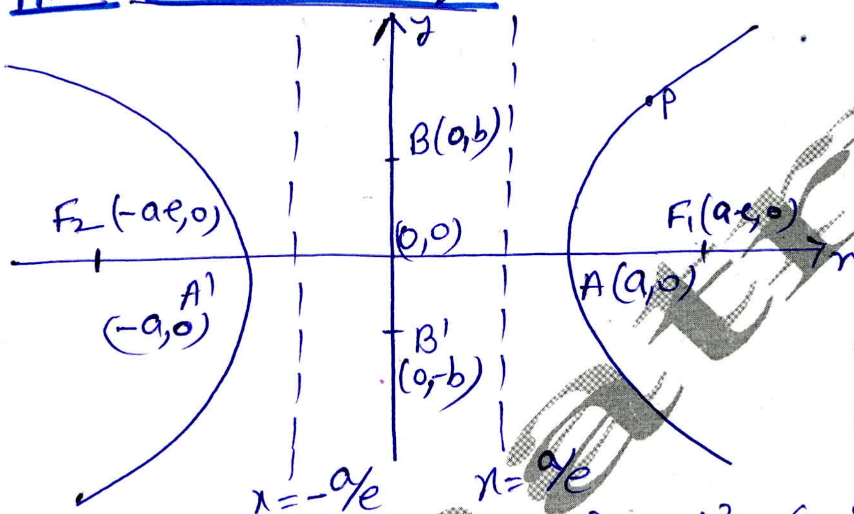
H-1



$P_1F_2 - P_1F_1 = P_2F_1 - P_2F_2 = \text{const.}$
= length of Transverse axis

The hyperbola is the set of all points in a plane, the difference of whose distance from two fixed points in the plane is a constant.

H-2 Standard Eq^n



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$b^2 = a^2(e^2-1)$

Transverse Axis $AA' = 2a$

Conjugate Axis $BB' = 2b$

foci $(\pm ae, 0)$

Directrix $x = \pm \frac{a}{e}$

$PF_2 - PF_1 = 2a$

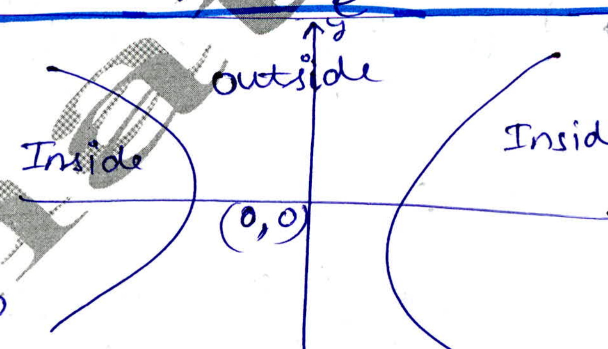
Latus Rectum length $= \frac{2b^2}{a} = \frac{4b^2}{2a} = \frac{(\text{Conjugate Axis})^2}{\text{Transverse Axis}} = 2e\left(ae - \frac{a}{e}\right)$

$= 2e$(dist. b/w focus and corresponding foot of directrix)

H-3 Eq^n of Hyperbola whose Axes are parallel to Coordinate Axes and centre is (h,k)

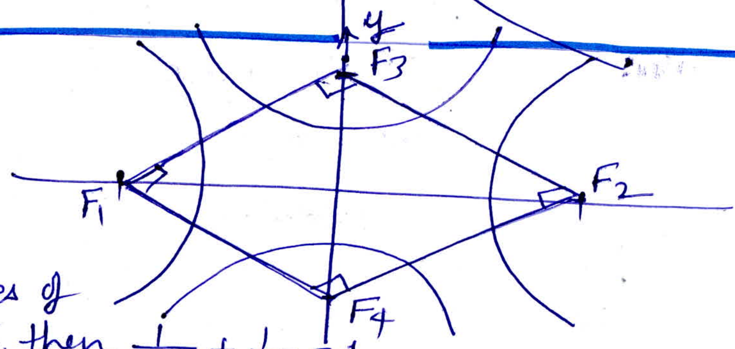
$\Rightarrow \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad (a>b)$, foci $(h \pm ae, k)$, Directrix $x = h \pm \frac{a}{e}$

H-4 Position of a point (h,k) wrt a Hyperbola



pt $P(x_1, y_1)$ lies inside, on or outside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ according to $\left(\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1\right) >, =, < 0$

H-5 Conjugate Hyperbola



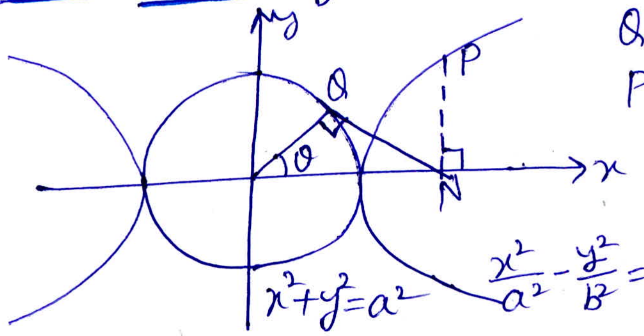
Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Conjugate Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

(i) If e_1 & e_2 are the eccentricities of a Hyperbola and its conjugate, then $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

(ii) The foci of a Hyperbola & its conjugate are concyclic & form the vertices of a square

H-6 Auxiliary Circle and Eccentric Angle



$Q(a\cos\theta, a\sin\theta)$ $\quad 0 \le \theta < 2\pi$

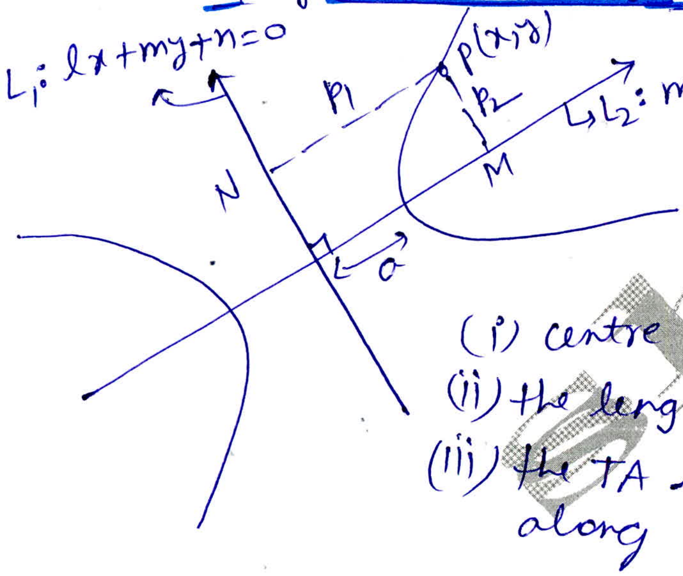
$P(a\sec\theta, b\tan\theta)$

θ is called the eccentric angle of point P. on the hyperbola.

H-7 Comparison of Hyperbola and its Conjugate Hyperbola

	Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	Conjugate Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
Centre	$(0,0)$	$(0,0)$
Length of TA	$2a$	$2b$
Length of CA	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Directrix	$x = \pm a/e$	$y = \pm b/e$
eccentricity	$b^2 = a^2(e^2-1)$	$a^2 = b^2(e^2-1)$
Latus Rectum	$2b^2/a$	$2a^2/b$
Parametric pt	$(a\sec\theta, b\tan\theta)$	$(\cancel{b\sec\theta}$ $a\tan\theta, b\sec\theta)$

H-8 Eqn of Hyperbola referred to two perpendicular lines



$$\Rightarrow \frac{p_1^2}{a^2} - \frac{p_2^2}{b^2} = 1$$

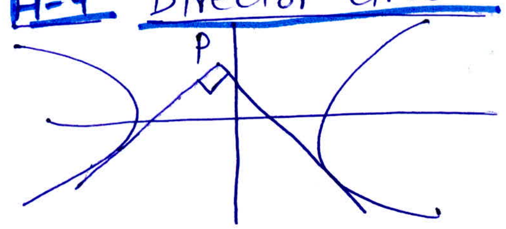
$$\Rightarrow \frac{\left(\frac{lx+my+n}{\sqrt{l^2+m^2}}\right)^2}{a^2} - \frac{\left(\frac{mx-ly+p}{\sqrt{m^2+l^2}}\right)^2}{b^2} = 1$$

(i) centre of Hyperbola is pt of intersection of $L_1=0$ & $L_2=0$

(ii) the length of TA & CA are $2a$ & $2b$ if $a>b$

(iii) the TA lies along $L_2=0$ and the CA lies along $L_1=0$.

H-9 Director Circle



$x^2+y^2=a^2-b^2$

$\Rightarrow a>b$, DC is real with finite radius.

$a=b$, DC is a point circle which is $(0,0)$

$a<b$, no real circle

H-10 Eq^n of Tangent

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

(i) Point form (x_1, y_1) : $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

(ii) Parametric form $(a\sec\theta, b\tan\theta)$: $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

(iii) Slope form (m) : $y=mx\pm\sqrt{a^2m^2-b^2}$

(iv) Eq^n of tangent at pt (x_1, y_1) to hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ is

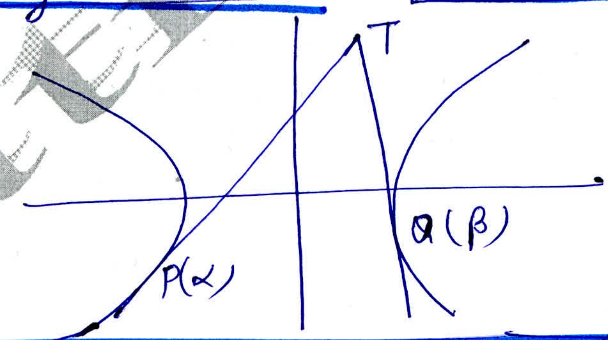
$$\frac{(x-h)(x_1-h)}{a^2}-\frac{(y-k)(y_1-k)}{b^2}=1$$

(v) the line $y=mx+c$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ if $c^2=a^2m^2-b^2$

H-11 Point of Intersection of tangents at Point $P(\alpha)$ and $Q(\beta)$

pt. of intersection is

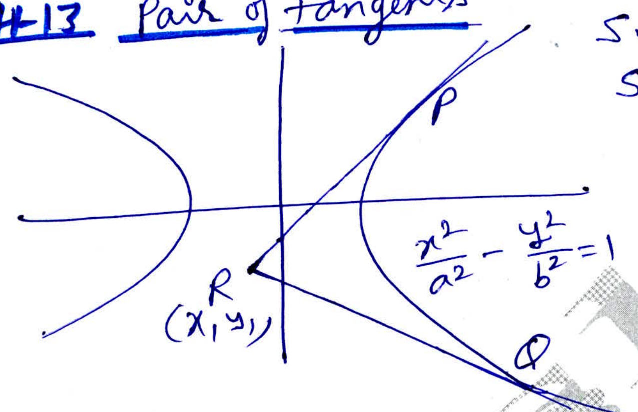
$$\left(a\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)},\ b\frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}\right)$$



H-12 Eq^n of chord joining $P(\alpha)$ and $Q(\beta)$

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right)-\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha+\beta}{2}\right)$$

H-13 Pair of tangents



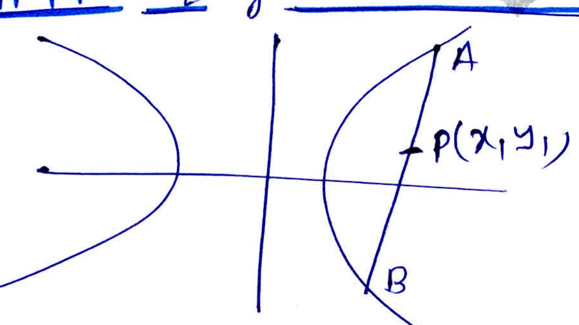
$SS_1=T^2$

$S : \frac{x^2}{a^2}-\frac{y^2}{b^2}-1$

$S_1 : \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$

$T : \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$

H-14 Eq^n of chord whose mid pt is $P(x_1, y_1)$

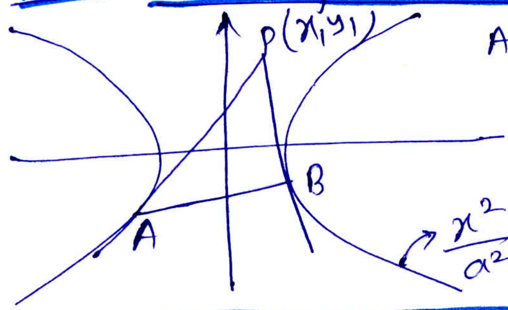


$AB : T=S_1$

where $T : \frac{xx_1}{a^2}-\frac{yy_1}{b^2}-1$

$S_1 : \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1$

H-15 Chord of Contact

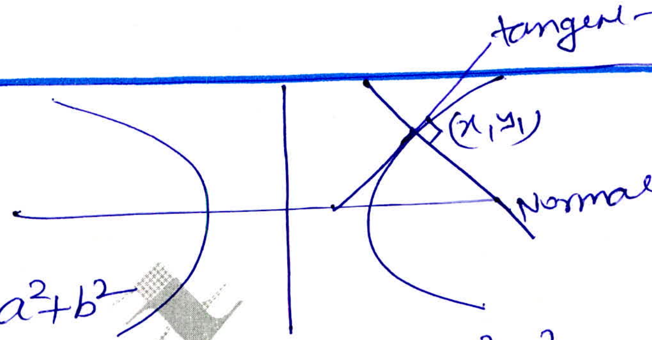


AB : $T = 0$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = 0$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

H-16 Eq^n of Normal



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(i) Point form (x_1, y_1) : $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2+b^2$

(ii) Parametric form $(a\sec\theta, b\tan\theta)$: $ax\cos\theta + by\cot\theta = a^2+b^2$

H-17 Properties of Normal

(i) Normal other than transverse axis never passes thru focus.

(ii) Locus of feet of the perpendiculars drawn from focus upon any tangent of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is its auxiliary circle ie $x^2+y^2 = a^2$

(iii) The product of perpendiculars drawn ~~fo~~ from foci upon any tangent of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is b^2.

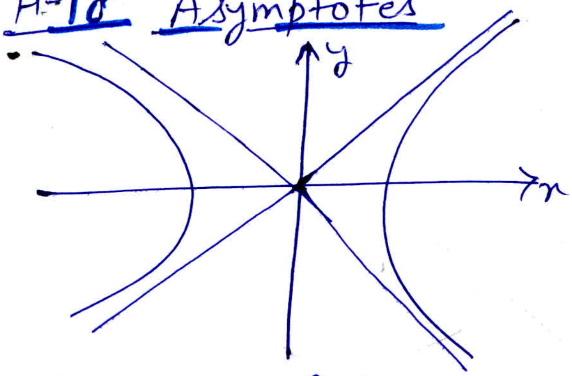
(iv) The portion of the tangent between the point of contact and the point where it meets the directrix subtends a right angle at corresponding focus.

(v) The tangent and normal at any point of hyperbola bisect the angle between the focal radii.
Hence "an incoming light ray" aimed towards one focus is reflected from the outer surface of the hyperbola towards the other focus.

(vi) If an ellipse and a hyperbola have same foci, they cut at right angles at any of their common points.

(vii) The foci of the hyperbola and the points P & Q in which any tangent meets the tangents at the vertices are concyclic with PQ as diameter of the circle.

H-18 Asymptotes



Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Asymptote $y = \pm \frac{b}{a}x$

ie $\frac{x}{a} + \frac{y}{b} = 0$ & $\frac{x}{a} - \frac{y}{b} = 0$

Important Points

(i) If angle between the asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 2θ, then $e = \sec\theta$. Also acute angle between the asymptotes is $\tan^{-1}\left|\frac{2ab}{a^2-b^2}\right|$

(ii) A hyperbola and its conjugate have the same asymptote.

(iii) The asymptotes pass thru the centre of the hyperbola and the bisectors of the angle between the asymptotes are the axes of the hyperbola

(iv) The equation of pair of asymptotes differ from the equation of the hyperbola and by the conjugate hyperbola by some constant only.

(v) The asymptotes of a hyperbola are the diagonals of rectangle formed by the line drawn through the extremities of each axis parallel to the other axis.

(vi) For rectangular or equilateral hyperbola $a=b$. Then the asymptotes of the rectangular hyperbola $x^2 - y^2 = a^2$ are $y = \pm x$ which are at right angle.

(vii) If from any point on the asymptotes a straight line is drawn perpendicular to the transverse axis, the product of the segments of this line, intercepted between the point and curve is always equal to the square of the semi-conjugate axis.

(viii) Perpendicular from the foci on either asymptote meet it at the same point as the corresponding directrix and the common points of intersection lie on the auxiliary circle.

(ix) If the asymptotes of a rectangular hyperbola are $x = \alpha$ and $y = \beta$, then its eqⁿ is $(x-\alpha)(y-\beta) = c^2$.

H-19 Rectangular Hyperbola Referred to its Asymptotes as the Axes of Coordinates

$xy = c^2$, $e = \sqrt{2}$

Asymptotes $x = 0$, $y = 0$

TA : $y = x$, CA : $y = -x$

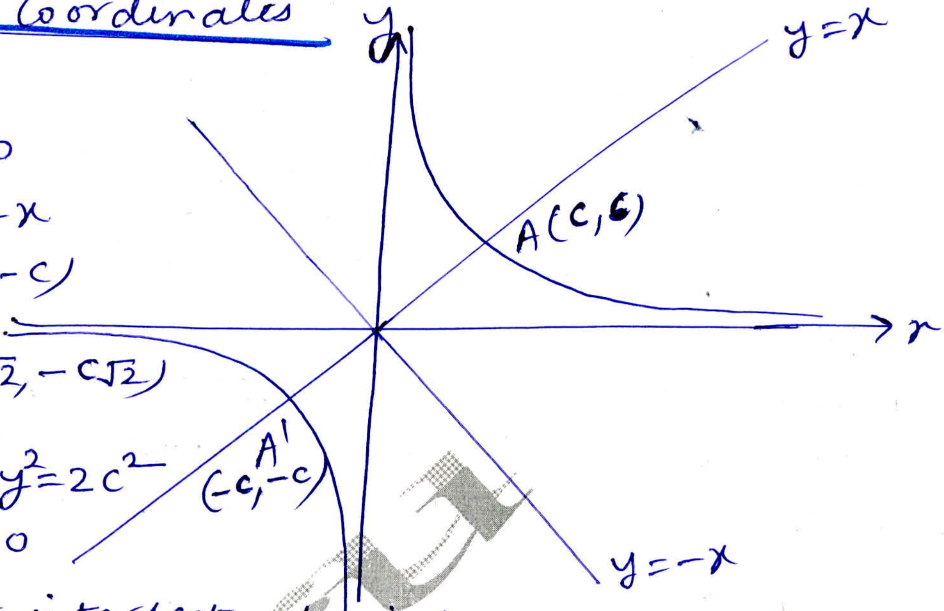
Vertex $A(c, c)$, $A'(-c, -c)$

Foci $(c\sqrt{2}, c\sqrt{2})$ & $(-c\sqrt{2}, -c\sqrt{2})$

length of LR $= 2\sqrt{2}c$

Auxiliary Circle $x^2 + y^2 = 2c^2$

Director Circle, $x^2 + y^2 = 0$



$x^2 - y^2 = 1$ and $xy = c^2$ intersect at right angle

Parametric pt on $xy = c^2$ is $\left(ct, \frac{c}{t}\right)$ $t \in R \sim \{0\}$

Eq^n of tangent at 't' : $x + yt^2 - 2ct = 0$

Eq^n of Normal at 't' : $xt^3 - yt - ct^4 + c = 0$

Eq^n of tangent at (x_1, y_1) : $xy_1 + yx_1 = 2c^2$

Eq^n of Normal at (x_1, y_1) : $xx_1 - yy_1 = x_1^2 - y_1^2$

H-20 Concyclic Points on $xy = c^2$

If a circle and the Rectangular hyperbola $xy = c^2$ meet at the four points t_1, t_2, t_3, t_4 then

(i) $t_1 t_2 t_3 t_4 = 1$

(ii) the centre of the mean position of the four points bisects the distance between the centres of the two curves.